Image Formation and Cameras

Introduction to Computer Vision
CSE 152
Lecture 4
Announcements

• Read Trucco & Verri: pp. 15-40
• HW1 will be on web site tomorrow or Saturday.
• Irfanview: http://www.irfanview.com/ is a good windows utility for manipulating images. Try xv for linux.
Pinhole Camera: Perspective projection

- Abstract camera model - box with a small hole in it
Geometric Aspects of Perspective Projection

• Points project to points
• Lines project to lines
• Angles & distances (or ratios) are NOT preserved under perspective
• Vanishing point
The equation of projection

Cartesian coordinates:

- We have, by similar triangles, that $(x, y, z) \rightarrow (f \frac{x}{z}, f \frac{y}{z}, -f)$
- Ignore the third coordinate, and get $(x, y, z) \rightarrow (f \frac{x}{z}, f \frac{y}{z})$
Euclidean -> Homogenous-> Euclidean

In 2-D
• Euclidean -> Homogenous: \((x, y) \rightarrow \lambda \ (x,y,1)\)
  (can just take \(\lambda = 1\))
• Homogenous -> Euclidean: \((x, y, z) \rightarrow (x/z, y/z)\)

In 3-D
• Euclidean -> Homogenous: \((x, y, z) \rightarrow \lambda(x,y,z,1)\)
  (can just take \(\lambda = 1\))
• Homogenous -> Euclidean: \((x, y, z, w) \rightarrow (x/w, y/w, z/w)\)
The camera matrix

Turn

\[(x, y, z) \rightarrow (f \frac{x}{z}, f \frac{y}{z})\]

into homogenous coordinates

- HC’s for 3D point are \((X, Y, Z, 1)\)
- HC’s for point in image are \((U, V, W)\)
Affine Camera Model

- Take Perspective projection equation, and perform Taylor Series Expansion about (some point $(x_0,y_0,z_0)$).
- Drop terms of higher order than linear.
- Resulting expression is called affine camera model.

- Properties
  - Pts. map to pts, lines map to lines
  - Parallel lines map to parallel lines (no vanishing point – at infinity)
  - Ratios of distance/angles preserved
Orthographic projection

Start with affine camera model, and take Taylor series about 
\((x_0, y_0, z_o) = (0, 0, z_0)\) – a point on optical axis

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
X \\
Y \\
Z \\
1
\end{bmatrix} =
\begin{bmatrix}
x' \\
y'
\end{bmatrix}
\]

Depth (z) is lost
Coordinate Changes: Pure Translations
No rotation (e.g., $i_A = i_b$ etc)

$O_B P = O_B O_A + O_A P$, $B P = A P + B O_A$
Coordinate Changes: Pure Rotations

\[
\overrightarrow{OP} = \begin{bmatrix}
A_x \\
A_y \\
A_z
\end{bmatrix} = \begin{bmatrix}
A_x \\
A_y \\
A_z
\end{bmatrix} = \begin{bmatrix}
B_x \\
B_y \\
B_z
\end{bmatrix}
\]

\[\Rightarrow \overrightarrow{BP} = B R^A P\]

A rotation matrix \( R \) has the following properties:
- Its inverse is equal to its transpose \( R^{-1} = R^T \)
- Its determinant is equal to 1: \( \det(R) = 1 \).

Or equivalently:
- Rows (or columns) of \( R \) form a right-handed orthonormal coordinate system.
Coordinate Changes: Rigid Transformations
both translation and rotation

\[ \mathbf{B} \mathbf{P} = \mathbf{B} \mathbf{R} \mathbf{A} \mathbf{P} + \mathbf{B} \mathbf{O}_A \]
Rigid Transformations as Mappings: Rotation about the $k$ Axis

$$FP' = \mathcal{R}^{F}P,$$

where

$$\mathcal{R} = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} = \text{rot}(k, \theta)$$

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Intro Computer Vision
Rotation: Homogenous Coordinates

- About z (k in prior fig) in axis

\[
\begin{pmatrix}
    x' \\
    y' \\
    z' \\
    1
\end{pmatrix} =
\begin{bmatrix}
    \cos \theta & -\sin \theta & 0 & 0 \\
    \sin \theta & \cos \theta & 0 & 0 \\
    0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 1
\end{bmatrix}
\begin{pmatrix}
    x \\
    y \\
    z \\
    1
\end{pmatrix}
\]
Rotation

About x axis:

\[
\begin{pmatrix}
  x' \\
  y' \\
  z' \\
  1
\end{pmatrix}
= \begin{pmatrix}
  1 & 0 & 0 & 0 \\
  0 & \cos \theta & -\sin \theta & 0 \\
  0 & \sin \theta & \cos \theta & 0 \\
  0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
  x \\
  y \\
  z \\
  1
\end{pmatrix}
\]

About y axis:

\[
\begin{pmatrix}
  x' \\
  y' \\
  z' \\
  1
\end{pmatrix}
= \begin{pmatrix}
  \cos \theta & 0 & \sin \theta & 0 \\
  0 & 1 & 0 & 0 \\
  -\sin \theta & 0 & \cos \theta & 0 \\
  0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
  x \\
  y \\
  z \\
  1
\end{pmatrix}
\]
Roll-Pitch-Yaw
\[ R = rot(\hat{i}, \alpha)rot(\hat{j}, \beta)rot(\hat{k}, \varphi) \]

Euler Angles
\[ R = rot(\hat{k}'', \alpha)rot(\hat{j}', \beta)rot(\hat{k}, \varphi) \]
Rotation

- About \((k_x, k_y, k_z)\), a unit vector on an arbitrary axis (Rodrigues Formula)

\[
\begin{pmatrix}
  x' \\
  y' \\
  z' \\
  1
\end{pmatrix} =
\begin{pmatrix}
  k_x k_x (1-c) + c & k_x k_y (1-c) - k_z s & k_x k_z (1-c) + k_y s & 0 \\
  k_y k_x (1-c) + k_z s & k_y k_y (1-c) + c & k_y k_z (1-c) - k_x s & 0 \\
  k_z k_x (1-c) - k_y s & k_z k_y (1-c) - k_x s & k_z k_z (1-c) + c & 0 \\
  0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z \\
1
\end{pmatrix}
\]

where \( c = \cos \theta \) & \( s = \sin \theta \)
Block Matrix Multiplication

\[
A = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix} \quad B = \begin{bmatrix}
B_{11} & B_{12} \\
B_{21} & B_{22}
\end{bmatrix}
\]

What is \(AB\)?

\[
AB = \begin{bmatrix}
A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\
A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22}
\end{bmatrix}
\]

Homogeneous Representation of Rigid Transformations

\[
\begin{bmatrix}
^B P \\
1
\end{bmatrix} = \begin{bmatrix}
^B R & ^B O_A \\
0^T & 1
\end{bmatrix} \begin{bmatrix}
^A P \\
1
\end{bmatrix} = \begin{bmatrix}
^B T & ^A P \\
1 & 1
\end{bmatrix}
\]

Transformation represented by 4 by 4 Matrix
Camera parameters

• Issue
  – camera may not be at the origin, looking down the z-axis
    • extrinsic parameters (Rigid Transformation)
  – one unit in camera coordinates may not be the same as one unit in world coordinates
    • intrinsic parameters - focal length, principal point, aspect ratio, angle between axes, etc.

\[
\begin{pmatrix}
U \\
V \\
W
\end{pmatrix} = \begin{pmatrix}
\text{Transformation} & 1 & 0 & 0 & 0 \\
\text{representing} & 0 & 1 & 0 & 0 \\
\text{intrinsic parameters} & 0 & 0 & 1 & 0 \\
\end{pmatrix} \begin{pmatrix}
X \\
Y \\
Z \\
T
\end{pmatrix}
\]

3 x 3 4 x 4
Given $n$ points $P_1, \ldots, P_n$ with known positions and their images $p_1, \ldots, p_n$, estimate intrinsic and extrinsic camera parameters.

- See Text book for how to do it.
What about light?
Getting more light – Bigger Aperture
Limits for pinhole cameras

2.18 DIFFRACTION LIMITS THE QUALITY OF PINHOLE OPTICS. These three images of a bulb filament were made using pinholes with decreasing size. (A) When the pinhole is relatively large, the image rays are not properly converged, and the image is blurred. (B) Reducing the size of the pinhole improves the focus. (C) Reducing the size of the pinhole further worsens the focus, due to diffraction. From Ruechardt, 1958.
Pinhole Camera Images with Variable Aperture

2 mm  1 mm

.6 mm  .35 mm

.15 mm  .07 mm
The reason for lenses
Thin Lens

- Rotationally symmetric about optical axis.
- Spherical interfaces.
Thin Lens: Center

- All rays that enter lens along line pointing at O emerge in same direction.
Thin Lens: Focus

Parallel lines pass through the focus, F
Thin Lens: Image of Point

All rays passing through lens and starting at \( P \) converge upon \( P' \)
Thin Lens: Image of Point

\[ \frac{1}{z'} - \frac{1}{z} = \frac{1}{f} \]
Thin Lens: Image Plane

Image Plane

A price: Whereas the image of $P$ is in focus, the image of $Q$ isn’t.
Thin Lens: Aperture

- Smaller Aperture -> Less Blur
- Pinhole -> No Blur
Deviations from the lens model

Deviations from this ideal are *aberrations*

*Two types*

1. geometrical
   - spherical aberration
   - astigmatism
   - distortion
   - coma

2. chromatic

Aberrations are reduced by combining lenses

*Compound lenses*
Spherical aberration

rays parallel to the axis do not converge

outer portions of the lens yield smaller focal lengths
Distortion

magnification/focal length different for different angles of inclination

Can be corrected! (if parameters are known)
Chromatic aberration

Index of refraction of lens depends on wavelength of light
Chromatic aberration

rays of different wavelengths focused in different planes

cannot be removed completely

sometimes *achromatization* is achieved for more than 2 wavelengths
Vignetting in Compound Lenses
Radiometry, Lighting, Intensity
Lighting

• Applied lighting can be represented as a function on the 4-D ray space (radiances)
• Special light sources
  – Point sources
  – Distant point sources
  – Strip sources
  – Area sources
• Common to think of lighting at infinity (a function on the sphere, a 2-D space)
**Radiance**

- Power traveling at some point in a specified direction, per unit area perpendicular to the direction of travel, per unit solid angle.

\[ L = \frac{P}{(dA \cos \theta)d\omega} \]

- Units: watts per square meter per steradian: \( \text{w/(m}^2\text{sr}^1) \)

**Irradiance**

- How much light is arriving at a surface?

- Irradiance -- power per unit area: W/cm\(^2\)

- Total power arriving at the surface is given by adding irradiance over all incoming angles.

\[ P = \int \text{irradiance} \, dA \]