Image Formation and Cameras

Introduction to Computer Vision
CSE 152
Lecture 3
Announcements

• Assignment 0: due today
• Office Hour: Thursday 12:30-1:30
• Read Trucco & Verri: pp. 15-40
Image Formation: Outline

• Factors in producing images
• Projection
• Perspective
• Vanishing points
• Orthographic
• Lenses
• Sensors
• Quantization/Resolution
• Illumination
• Reflectance
Earliest Surviving Photograph

• First photograph on record, “la table service” by Nicephore Niepce in 1822.
• Note: First photograph by Niepce was in 1816.
How Cameras Produce Images

• Basic process:
  – photons hit a detector
  – the detector becomes charged
  – the charge is read out as brightness

• Sensor types:
  – CCD (charge-coupled device)
    • high sensitivity
    • high power
    • cannot be individually addressed
    • blooming
  – CMOS
    • most common
    • simple to fabricate (cheap)
    • lower sensitivity, lower power
    • can be individually addressed
Images are two-dimensional patterns of brightness values.

They are formed by the projection of 3D objects.
Effect of Lighting: Monet
Change of Viewpoint: Monet

Haystack at Chailly at Sunrise (1865)
Pinhole Camera: **Perspective projection**

- Abstract camera model - box with a small hole in it
Camera Obscura

"When images of illuminated objects ... penetrate through a small hole into a very dark room ... you will see [on the opposite wall] these objects in their proper form and color, reduced in size ... in a reversed position, owing to the intersection of the rays".

Da Vinci

http://www.acmi.net.au/AIC/CAMERA_OBSCURA.html (Russell Naughton)
• Used to observe eclipses (eg., Bacon, 1214-1294)
• By artists (eg., Vermeer).
Jetty at Margate England, 1898.

http://brightbytes.com/cosite/collection2.html (Jack and Beverly Wilgus)
Distant objects are smaller

(Forsyth & Ponce)
Geometric properties of projection

- Points go to points
- Lines go to lines
- Planes go to whole image or half-plane
- Polygons go to polygons
- Angles & distances not preserved

- Degenerate cases:
  - line through focal point yields point
  - plane through focal point yields line
Parallel lines meet in the image

• vanishing point
Take out paper and pencil
Draw a horizon line.

Make a vanishing point.

Draw a square or rectangle.

Draw orthogonals from shape corners to vanishing point.

Draw a horizontal line to end your form.

Draw a vertical line to make the form's side.

Erase the orthogonals.

Draw another form!

Add windows and doors.
Add windows and doors.
Vanishing points

To different directions correspond different vanishing points.
Vanishing Points
The equation of projection

Cartesian coordinates:

- We have, by similar triangles, that $(x, y, z) \rightarrow \left( \frac{fx}{z}, \frac{fy}{z}, -f \right)$
- Ignore the third coordinate, and get $(x, y, z) \rightarrow \left( \frac{x}{z}, \frac{y}{z} \right)$
A Digression

Homogenous Coordinates
and
Camera Matrices
Homogenous coordinates

- Our usual coordinate system is called a Euclidean or affine coordinate system.

- Rotations, translations and projection in Homogenous coordinates can be expressed linearly as matrix multiplies.
Projective Geometry

- Axioms of Projective Plane
  1. Every two distinct points define a line
  2. Every two distinct lines define a point (intersect at a point)
  3. There exist three points, A, B, C such that C does not lie on the line defined by A and B.

- Different than Euclidean (affine) geometry

- Projective plane is “bigger” than affine plane – includes “line at infinity”

\[
\text{Projective Plane} = \text{Affine Plane} + \text{Line at Infinity}
\]
Homogenous coordinates
A way to represent points in a projective space

1. Add an extra coordinate
e.g., \((x,y) \rightarrow (x,y,1) = (u,v,w)\)

2. Impose equivalence relation such that \((\lambda \not= 0)\)
\((u,v,w) \approx \lambda*(u,v,w)\)
i.e., \((x,y,1) \approx (\lambda x, \lambda y, \lambda)\)

3. “Point at infinity” – zero for last coordinate
e.g., \((x,y,0)\)

- Why do this?
  - Possible to represent points “at infinity”
    - Where parallel lines intersect
    - Where parallel planes intersect
  - Possible to write the action of a perspective camera as a matrix
Euclidean -> Homogenous-> Euclidean

In 2-D

• Euclidean -> Homogenous: (x, y) -> k (x,y,1)
• Homogenous -> Euclidean: (u,v,w) -> (u/w, v/w)

In 3-D

• Euclidean -> Homogenous: (x, y, z) -> k (x,y,z,1)
• Homogenous -> Euclidean: (x, y, z, w) -> (x/w, y/w, z/w)
The camera matrix

\[(x, y, z) \rightarrow (f \frac{x}{z}, f \frac{y}{z})\]

Turn this expression into homogenous coordinates

- HC’s for 3D point are (X,Y,Z,T)
- HC’s for point in image are (U,V,W)

\[
\begin{pmatrix}
U \\
V \\
W
\end{pmatrix} =
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1/f & 0
\end{pmatrix}
\begin{pmatrix}
X \\
Y \\
Z \\
T
\end{pmatrix}
\]

Perspective Camera Matrix
A 3x4 matrix
What is the intersection of two lines in a plane?

A Point
Do two lines in the plane always intersect at a point?

No, Parallel lines don’t meet at a point.
Can the perspective image of two parallel lines meet at a point?

YES
Projective geometry provides an elegant means for handling these different situations in a unified way and homogenous coordinates are a way to represent entities (points & lines) in projective spaces.
End of the Digression
Simplified Camera Models

- Perspective Projection
- Affine Camera Model
- Scaled Orthographic Projection
- Orthographic Projection
Affine Camera Model

- Take Perspective projection equation, and perform Taylor Series Expansion about (some point $(x_0, y_0, z_0)$).
- Drop terms of higher order than linear.
- Resulting expression is affine camera model.
• Perspective

\[
\begin{bmatrix}
u \\
v
\end{bmatrix} = f\frac{1}{z} \begin{bmatrix}x \\
y
\end{bmatrix}
\]

• Assume that \( f = 1 \), and perform a Taylor series expansion about \((x_0, y_0, z_0)\)

\[
\begin{bmatrix}u \\
v
\end{bmatrix} = \frac{1}{z_0} \begin{bmatrix}x_0 \\
y_0
\end{bmatrix} - \frac{1}{z_0^2} \begin{bmatrix}x_0 \\
y_0
\end{bmatrix}(z - z_0) + \frac{1}{z_0} \begin{bmatrix}1 \\
0
\end{bmatrix}(x - x_0)
\]

\[
+ \frac{1}{z_0} \begin{bmatrix}0 \\
1
\end{bmatrix}(y - y_0) + \frac{1}{2} \frac{2}{z_0^3} \begin{bmatrix}x_0 \\
y_0
\end{bmatrix}(z - z_0)^2 + \ldots
\]

• Dropping higher order terms and regrouping.

\[
\begin{bmatrix}u \\
v
\end{bmatrix} \approx \frac{1}{z_0} \begin{bmatrix}x_0 \\
y_0
\end{bmatrix} + \begin{bmatrix}1/z_0 & 0 & -x_0/z_0^2 \\
0 & 1/z_0 & -y_0/z_0^2
\end{bmatrix} \begin{bmatrix}x \\
y \\
z
\end{bmatrix} = Ap + b
\]
\[
\begin{bmatrix}
  u \\
  v \\
  w
\end{bmatrix} \approx \frac{1}{z_0} \begin{bmatrix} x_0 \\
  y_0 \end{bmatrix} + \begin{bmatrix} 1/z_0 & 0 & -x_0/z_0^2 \\
  0 & 1/z_0 & -y_0/z_0^2 \end{bmatrix} \begin{bmatrix} x \\
  y \\
  z \end{bmatrix} = Ap + b
\]

Rewrite Affine camera model in terms of Homogenous Coordinates

\[
\begin{bmatrix}
  u \\
  v \\
  w
\end{bmatrix} \approx \begin{bmatrix} 1/z_0 & 0 & -x_0/z_0^2 & x_0/z_0 \\
  0 & 1/z_0 & -y_0/z_0^2 & y_0/z_0 \\
  0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix} x \\
  y \\
  z \\
  1 \end{bmatrix}
\]
Consider doing an expansion about a point $X_0=0$, $Y_0 = 0$

\[
\begin{bmatrix}
u \\
v \\
w \end{bmatrix} \approx \begin{bmatrix} 1/z_0 & 0 & -x_0/z_0^2 & x_0/z_0 \\ 0 & 1/z_0 & -y_0/z_0^2 & y_0/z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}
\]
The projection matrix for orthographic projection

\[
\begin{pmatrix}
U \\
V \\
W
\end{pmatrix} = \begin{pmatrix}
1/z_0 & 0 & 0 & 0 \\
0 & 1/z_0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
X \\
Y \\
Z \\
T
\end{pmatrix}
\]

Parallel lines project to parallel lines
Ratios of distances are preserved under orthographic
Orthographic projection

Starting with Affine camera mode

Take Taylor series about \((0, 0, z_0)\) – a point on optical axis

\[
\begin{bmatrix}
u \\
v
\end{bmatrix} = \frac{1}{z_0} \begin{bmatrix} x \\
y\end{bmatrix}
\]
Other camera models

- Generalized camera – maps points lying on rays and maps them to points on the image plane.

Omnican (hemispherical)  Light Probe (spherical)
Some Alternative “Cameras”
What if camera coordinate system differs from object coordinate system
Euclidean Coordinate Systems

\[
\begin{align*}
\vec{x} &= \overrightarrow{OP}.\vec{i} \\
\vec{y} &= \overrightarrow{OP}.\vec{j} \\
\vec{z} &= \overrightarrow{OP}.\vec{k}
\end{align*}
\iff
\overrightarrow{OP} = x\vec{i} + y\vec{j} + z\vec{k}
\iff
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\]
Coordinate Changes: Pure Translations
No rotation (e.g., $i_A = i_b$ etc)

$O_B P = O_B O_A + O_A P$, $B P = A P + B O_A$
Rotation Matrix

\[ (A) \]

\[ (B) \]

\[ B_R^A = \begin{bmatrix} i_A \cdot i_B & j_A \cdot i_B & k_A \cdot i_B \\ i_A \cdot j_B & j_A \cdot j_B & k_A \cdot j_B \\ i_A \cdot k_B & j_A \cdot k_B & k_A \cdot k_B \end{bmatrix} = \begin{bmatrix} A_i^T \\ A_j^T \\ A_k^T \end{bmatrix} = \begin{bmatrix} B_i & B_j & B_k \end{bmatrix} \]
A rotation matrix $R$ has the following properties:

- Its inverse is equal to its transpose $R^{-1} = R^T$
- Its determinant is equal to 1: $\det(R) = 1$.

Or equivalently:

- Rows (or columns) of $R$ form a right-handed orthonormal coordinate system.
Coordinate Changes: Pure Rotations

\[
\overrightarrow{OP} = \begin{bmatrix}
i_A & j_A & k_A
\end{bmatrix}
\begin{bmatrix}
A_x \\
A_y \\
A_z
\end{bmatrix}
= \begin{bmatrix}
i_B & j_B & k_B
\end{bmatrix}
\begin{bmatrix}
B_x \\
B_y \\
B_z
\end{bmatrix}
\]

\[\Rightarrow \quad ^B P = ^A R ^A P\]
Coordinate Changes: Rigid Transformations

\[
\begin{align*}
{B} P &= {B} R \ A P + {B} O_A \\
\end{align*}
\]