Stereo Vision II

Introduction to Computer Vision
CSE 152
Lecture 14
Announcements

• Midterm was returned on Tuesday

• Next HW assigned tomorrow
Stereo Vision Outline

- Offline: Calibrate cameras & determine “epipolar geometry”
- Online
  1. Acquire stereo images
  2. Rectify images to convenient epipolar geometry
  3. Establish correspondence
  4. Estimate depth
BINOCULAR STEREO SYSTEM

Estimating Depth

DISPARITY

\( (X_L - X_R) \)

\[
Z = \left( \frac{f}{X_L} \right) X \\
Z = \left( \frac{f}{X_R} \right) (X-d)
\]

\[
\left( \frac{f}{X_L} \right) X = \left( \frac{f}{X_R} \right) (X-d) \\
X = \frac{(X_L d)}{(X_L - X_R)}
\]

\[
X = \frac{d X_L}{(X_L - X_R)}
\]

\[
Z = \frac{d f}{(X_L - X_R)}
\]

(Adapted from Hager)
Reconstruction: General 3-D case

- **Linear Method:** find $P$ such that

  \[ \begin{align*}
    p \times M P &= 0 \\
    p' \times M' P &= 0
  \end{align*} \]

  \[ \iff \left( \begin{bmatrix} p_x \\ M \\ p'_x \end{bmatrix} \begin{bmatrix} M' \\ 0 \end{bmatrix} \right) P = 0 \]

- **Non-Linear Method:** find $Q$ minimizing

  \[ d^2(p, q) + d^2(p', q') \]
Random Dot Stereograms
• Potential matches for $p$ have to lie on the corresponding epipolar line $l'$. 

• Potential matches for $p'$ have to lie on the corresponding epipolar line $l$. 
• Epipolar Plane
• Baseline

• Epipoles

• Epipolar Lines
Family of epipolar Planes

Family of planes $\pi$ and lines $l$ and $l'$
Intersection in $e$ and $e'$
Epipolar Constraint: Calibrated Case

\[ \vec{O}p \cdot [\vec{O}O' \times \vec{O}'p'] = 0 \quad \Rightarrow \quad \vec{p} \cdot [t \times (\mathcal{R} \vec{p}')] = 0 \quad \text{with} \quad \begin{cases} \vec{p} = (u, v, 1)^T \\ \vec{p}' = (u', v', 1)^T \\ \mathcal{M} = (\text{Id} \quad \vec{0}) \\ \mathcal{M}' = (\mathcal{R}^T, -\mathcal{R}^T \vec{t}) \end{cases} \]

Essential Matrix
(Longuet-Higgins, 1981)

\[ \vec{p}^T \mathcal{E} \vec{p}' = 0 \quad \text{with} \quad \mathcal{E} = [t_x] \mathcal{R} \]

where \[ [t_x] = \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} \]
Properties of the Essential Matrix

\[ p^T \mathcal{E} p' = 0 \quad \text{with} \quad \mathcal{E} = [t_x] \mathcal{R} \]

- \( \mathcal{E} p' \) is the epipolar line associated with \( p' \).
- \( \mathcal{E}^T p \) is the epipolar line associated with \( p \).
- \( \mathcal{E} e' = 0 \) and \( \mathcal{E}^T e = 0 \).
- \( \mathcal{E} \) is singular.
- \( \mathcal{E} \) has two equal non-zero singular values (Huang and Faugeras, 1989).
Calibration

Determine intrinsic parameters and extrinsic relation of two cameras
The Eight-Point Algorithm (Longuet-Higgins, 1981)

\[
\begin{pmatrix}
(u, v, 1) \\
F_{11} & F_{12} & F_{13} \\
F_{21} & F_{22} & F_{23} \\
F_{31} & F_{32} & F_{33}
\end{pmatrix}
\begin{pmatrix}
u' \\
v' \\
1
\end{pmatrix} = 0
\]

\[
\begin{pmatrix}
F_{11} \\
F_{12} \\
F_{13} \\
F_{21} \\
F_{22} \\
F_{23} \\
F_{31} \\
F_{32} \\
F_{33}
\end{pmatrix} = 0
\]

Set \( F_{33} \) to 1

\[
\begin{pmatrix}
u_1u_1' & uv_1' & u_1' & v_1u_1' & v_1v_1' & v_1 & u_1' & v_1' \\
u_2u_2' & uv_2' & u_2' & v_2u_2' & v_2v_2' & v_2 & u_2' & v_2' \\
u_3u_3' & uv_3' & u_3' & v_3u_3' & v_3v_3' & v_3 & u_3' & v_3' \\
u_4u_4' & uv_4' & u_4' & v_4u_4' & v_4v_4' & v_4 & u_4' & v_4' \\
u_5u_5' & uv_5' & u_5' & v_5u_5' & v_5v_5' & v_5 & u_5' & v_5' \\
u_6u_6' & uv_6' & u_6' & v_6u_6' & v_6v_6' & v_6 & u_6' & v_6' \\
u_7u_7' & uv_7' & u_7' & v_7u_7' & v_7v_7' & v_7 & u_7' & v_7' \\
u_8u_8' & uv_8' & u_8' & v_8u_8' & v_8v_8' & v_8 & u_8' & v_8'
\end{pmatrix}
\begin{pmatrix}
F_{11} \\
F_{12} \\
F_{13} \\
F_{21} \\
F_{22} \\
F_{23} \\
F_{31} \\
F_{32} \\
F_{33}
\end{pmatrix} = -
\begin{pmatrix}
1 \\
1 \\
1 \\
1 \\
1 \\
1 \\
1 \\
1 \\
1
\end{pmatrix}
\]

Minimize:

\[
\sum_{i=1}^{n} (p_i^T \mathcal{F} p_i')^2
\]

under the constraint

\[
|\mathcal{F}|^2 = 1.
\]
Epipolar geometry example
Example: converging cameras
Example: forward motion
courtesy of Andrew Zisserman
Rectification

Given a pair of images, transform both images so that epipolar lines are scan lines.
Image pair rectification

simplify stereo matching by warping the images

Apply projective transformation so that epipolar lines correspond to horizontal scanlines

map epipole $e$ to $(1,0,0)$

try to minimize image distortion

Note that rectified images usually not rectangular
Rectification
Given a pair of images, transform both images so that epipolar lines are scan lines.

Input Images
Rectification

Given a pair of images, transform both images so that epipolar lines are scan lines.

Rectified Images

See Section 7.3.7 for specific method
Features on same epipolar line
Mobi: Stereo-based navigation
Epipolar correspondence

This version is feature-based: detect edges in 1-D signal, and use dynamic programming to find correspondences that minimize an error function.
Symbolic Map

LEFT SIDE

Hallway

RIGHT SIDE

Door

Door
A challenge: Multiple Interpretations

Each feature on left epipolar line match one and only one feature on right epipolar line.
Multiple Interpretations

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