

Logic – III

Introduction to Artificial Intelligence
CSE 150

Outline

Last Lecture

- Propositional Logic and Sound inference rules
- Introduction First order logic
 - Symbols, Variables, Functions, Predicates

This Lecture

- FOL Continued – Quantification
- Wumpus in FOL
- Inference in FOL
 - Unification
 - Generalized Modus Ponens
 - Forward and backward chaining

Reading

- Chapter 7

The Bad News

- The due date for PA2 is next Thursday

The Good News

- Midterm will be Thursday, May 10th

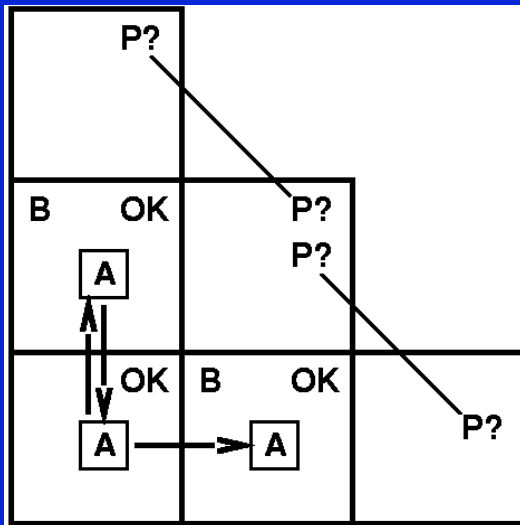
Inference rules

An inference rule is sound if the conclusion is true in all cases where the premises are true

α	Premise
<hr/>	
β	Conclusion

1. Modus Ponens
2. And Introduction
3. And Elimination
4. Double Negation

Knowledge Base in Wumpus World



$S_{i,j}$: Stench in cell i,j
 $B_{i,j}$: Breeze in cell i,j
 $W_{i,j}$: Wumpus in cell i,j

Percept Sentences

$$\neg S_{1,1} \quad \neg B_{1,1}$$

$$\neg S_{2,1} \quad B_{2,1}$$

$$\neg S_{1,2} \quad B_{1,2}$$

...

Environment Knowledge

(can be written before any sensing)

$$R_1: \neg S_{1,1} \Rightarrow \neg W_{1,1} \wedge \neg W_{2,1} \wedge \neg W_{1,2}$$

$$R_2: \neg S_{2,1} \Rightarrow \neg W_{1,1} \wedge \neg W_{2,1} \wedge \neg W_{2,2} \wedge \neg W_{3,1}$$

$$R_3: \neg B_{1,1} \Rightarrow \neg P_{1,1} \wedge \neg P_{2,1} \wedge \neg P_{1,2}$$

$$R_4: B_{2,1} \Rightarrow P_{1,1} \vee P_{2,1} \vee P_{2,2} \vee P_{3,1}$$

$$R_5: B_{1,2} \Rightarrow P_{1,1} \vee P_{1,2} \vee P_{2,2} \vee P_{1,3}$$

...

Inference in Wumpus World

Initial KB

Percept Sentences

$\neg S_{1,1}$ $\neg B_{1,1}$
 $\neg S_{2,1}$ $B_{2,1}$
 $\neg S_{1,2}$ $B_{1,2}$

...

Environment Knowledge

$R_1: \neg S_{1,1} \Rightarrow \neg W_{1,1} \wedge \neg W_{2,1} \wedge \neg W_{1,2}$
 $R_2: \neg S_{2,1} \Rightarrow \neg W_{1,1} \wedge \neg W_{2,1} \wedge \neg W_{2,2} \wedge \neg W_{3,1}$
 $R_3: \neg B_{1,1} \Rightarrow \neg P_{1,1} \wedge \neg P_{2,1} \wedge \neg P_{1,2}$
 $R_4: B_{2,1} \Rightarrow P_{1,1} \vee P_{2,1} \vee P_{2,2} \vee P_{3,1}$
 $R_5: B_{1,2} \Rightarrow P_{1,1} \vee P_{1,2} \vee P_{2,2} \vee P_{1,3}$

...

Some inferences:

Apply **Modus Ponens** to R_1

Add to KB

$\neg W_{1,1} \wedge \neg W_{2,1} \wedge \neg W_{1,2}$

Apply to this **AND-Elimination**

Add to KB

$\neg W_{1,1}$
 $\neg W_{2,1}$
 $\neg W_{1,2}$

Backus-Naur Form of Syntax

Sentence — *AtomicSentence*
| *Sentence* *Connective* *Sentence*
| *Quantifier* *Variable*, ... *Sentence*
| \neg *Sentence*
| (*Sentence*)

AtomicSentence — *Predicate*(*Term*, ...) | *Term* = *Term*

Term — *Function*(*Term*, ...)
| *Constant*
| *Variable*

Connective — \Rightarrow | \wedge | \vee | \Leftrightarrow

Quantifier — \forall | \exists

Constant — *A* | *X*₁ | *John* | ...

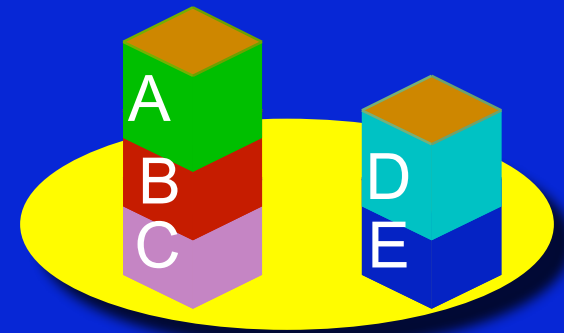
Variable — *a* | *x* | *s* | ...

Predicate — *Before* | *HasColor* | *Raining* | ...

Function — *Mother* | *LeftLegOf* | ...

Formalizing Knowledge

- The first step in formalizing knowledge is to construct a conceptualization, I.e., a representation of the objects in their world, and their relationships
 - Objects – Universe of discourse
 - Functions – Functional basis set
 - Relations (Predicates) – Relational basis set



A conceptualization is a triple

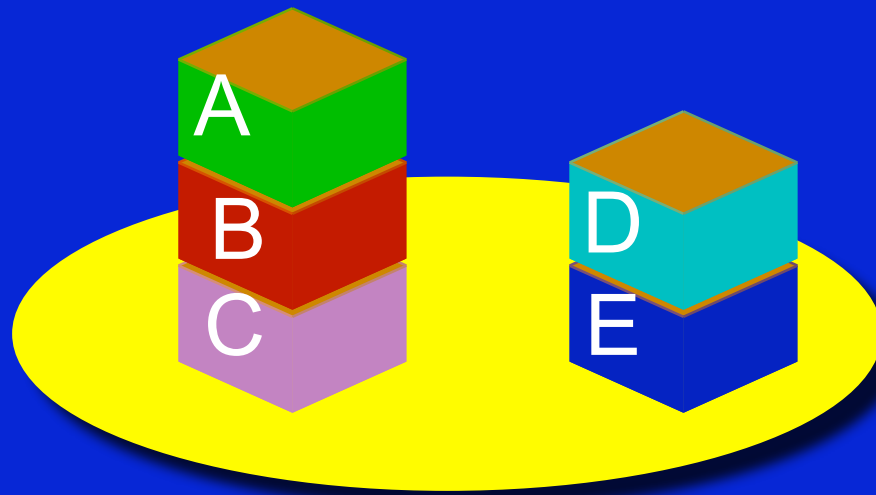
$\langle U \text{ of } D, \text{FBS}, \text{RBS} \rangle$

Block World: $\langle \{A, B, C, D, E\}, \{\text{HAT}, \dots\}, \{\text{ON}, \dots\} \rangle$

- Second step is to express conceptualization using Predicate Calculus

Sets

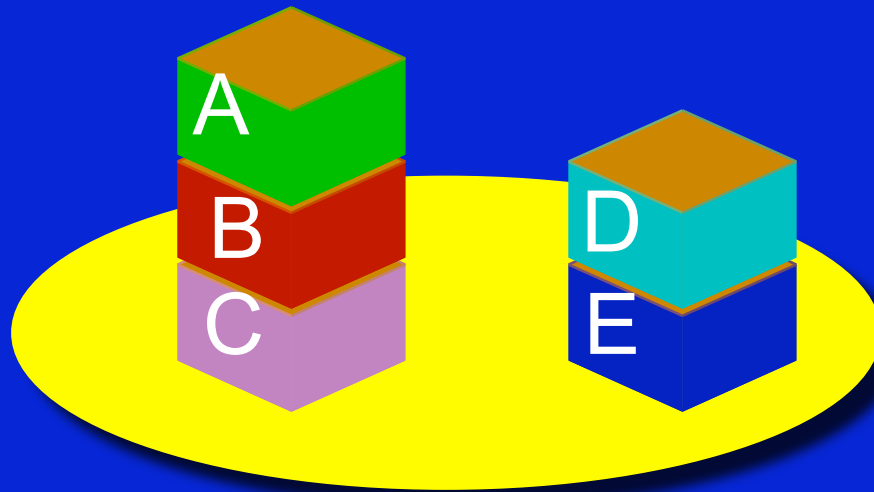
- The set of objects defines a “Universe of Discourse.”
- The objects are represented by Constant Symbols.



- e.g. For this blocks world, the universe of discourse is $\{A, B, C, D, E\}$

Relations

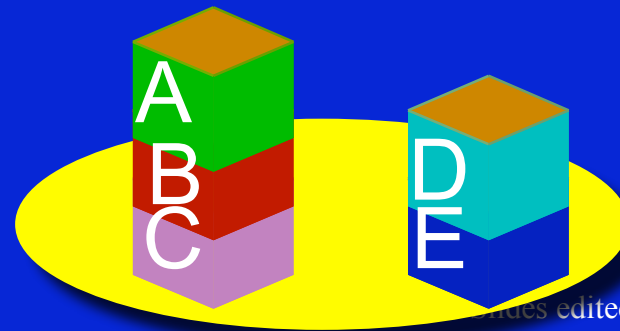
- Def: A **binary relation** is a set of ordered pairs
- Example: Consider set of blocks {A,B,C,D,E}



- The “ON” relation:
- $ON = \{ \langle A, B \rangle, \langle B, C \rangle, \langle D, E \rangle \}$
- The **predicate** $ON(A, B)$ means, $\langle A, B \rangle \in ON$
- $ON(A, B)$ is TRUE, but $ON(A, C)$ and $ON(C, D)$ are FALSE

Functions

- A function is a binary relation such that no two distinct members have the same first element
- If $\langle X, Y \rangle \in f$
 - X is an argument of f
 - Y is the value of f at X
 - Y is the image of f under X
- $Y = f(X)$
 - That is, $f(X)$ is equal to some symbol in Universe of Discourse
- $HAT = \{ \langle C, B \rangle, \langle B, A \rangle, \langle A, NIL \rangle, \langle E, D \rangle, \langle D, NIL \rangle \}$
 - $HAT(C) = B$
 - $HAT(E) = D$
 - $HAT(D) = NIL$



Universal quantification

- \forall <variables> <sentence>
For all (every) instances of the variables, the <sentence> is true
- Everyone at UCSD is smart
 $\forall x \text{ At}(x, \text{UCSD}) \Rightarrow \text{Smart}(x)$
- $\forall x P$ is equivalent to the **conjunction** of **instantiations** of P
(I.e., all x 's in universe of discourse)
At (Jonathan, UCSD) \Rightarrow Smart(Jonathan) \wedge
At (RussellImpagliazzo, UCSD) \Rightarrow Smart(RussellImpagliazzo) \wedge
At (BillClinton, UCSD) \Rightarrow Smart(BillClinton) \wedge
At (GeorgeWBush, UCSD) \Rightarrow Smart(GeorgWBush) \wedge ...
- Universe of discourse may be infinite, compare to propositional logic

Universal quantification cont.

Typically, \Rightarrow is the main connective with \forall

A common mistake is to use \wedge as the main connective with \forall

What does the following mean?

$\forall x \text{ At}(x, \text{UCSD}) \wedge \text{Smart}(x)$

“Everyone is at UCSD and everyone is smart”

Some more examples

- “Birds Fly”

$$\forall x \text{ Bird}(x) \Rightarrow \text{Fly}(x)$$

↑ ↑
predicate predicate

Some more examples

- “Birds Fly”

$$\forall x \text{ Bird}(x) \Rightarrow \text{Fly}(x)$$

↑ ↑
predicate predicate

- But what about penguins

$$\forall x \text{ Bird}(x) \wedge \neg \text{Penguin}(x) \Rightarrow \text{Fly}(x)$$

- “Fish Swim”

$$\forall x \text{ Fish}(x) \Rightarrow \text{Swim}(x)$$

Existential quantification

- \exists <variables> <sentence>
There exist instances of the <variables> such that the <sentence> is true
- Someone at Michigan is smart
 $\exists x \text{ At}(x, \text{Michigan}) \wedge \text{Smart}(x)$
- $\exists x P$ is equivalent to the **disjunction** of **instantiations** of P
(I.e., all x 's in universe of discourse)

$\text{At}(\text{Rodrigo}, \text{Michigan}) \wedge \text{Smart}(\text{Rodrigo}) \vee$
 $\text{At}(\text{Koditschek}, \text{Michigan}) \wedge \text{Smart}(\text{Koditschek}) \vee$
 $\text{At}(\text{Gore}, \text{Michigan}) \wedge \text{Smart}(\text{Gore}) \vee$
 $\text{At}(\text{Bush}, \text{Michigan}) \wedge \text{Smart}(\text{Bush}) \vee \dots$

Existential quantification cont.

Typically, \wedge is the main connective with \exists

A common mistake is to use \Rightarrow as the main connective with \exists

$\exists x \text{ At}(x, \text{UCSD}) \Rightarrow \text{Smart}(x)$

is true if there is anyone who is not at UCSD.

Another example

- “Some CSE150 student is sleeping”
 $\exists x \text{ CSE150Student}(x) \wedge \text{Sleeping}(x)$
- Note: There could be more than one sleeping student, but this says that there is at least one.
- To say “One CSE150 student is sleeping”
 $\exists x \text{ CSE150Student}(x) \wedge \text{Sleeping}(x) \wedge$
 $[\forall y (\text{CSE150}(y) \wedge \text{Sleeping}(y)) \Rightarrow (x=y)]$
- This is cumbersome, so sometimes a shorthand is used called the “uniqueness quantifier” $\exists! x$ I.e., there exists a unique x

Variables must be quantified

- In a well-formed formula (wff), all variables must be quantified
- The following are wffs
 - $\forall x \text{ At}(x, \text{UCSD}) \Rightarrow \text{Smart}(x)$
 - $\exists x \text{ At}(x, \text{Michigan}) \wedge \text{Smart}(x)$
- The following are not wffs
 - $\text{At}(x, \text{UCSD}) \Rightarrow \text{Smart}(x)$
 - $\text{At}(x, \text{Michigan}) \wedge \text{Smart}(x)$
- Occasionally, you'll see expressions with variables that are not quantified. In this case, there is an implicit universal quantifier on those variables.

Properties of quantifiers

- $\forall x \forall y$ is the same as $\forall y \forall x$ (why??)
- $\exists x \exists y$ is the same as $\exists y \exists x$ (why??)
- $\forall x \exists y$ is not the same as $\exists y \forall x$
- Example
 - Loves predicate: Loves(x,y) is true if x loves y
 - $\exists x \forall y \text{ Loves}(x,y)$
 - ``There is a person who loves everyone in the world''
 - $\forall y \exists x \text{ Loves}(x,y)$
 - ``Everyone in the world is loved by at least one person''

Quantifier Duality

Each quantifier (Universal or Existential) can be expressed using the other

$\forall x \text{ Likes}(x, \text{IceCream})$

- Everyone likes Ice Cream

$\neg \exists x \neg \text{ Likes}(x, \text{IceCream})$

[the dual]

- There does not exist anyone who does not like ice cream
- Nobody dislikes ice cream

$\exists x \text{ Likes}(x, \text{BrusselSprouts})$

- Someone likes Brussel sprouts

$\neg \forall x \neg \text{ Likes}(x, \text{BrusselSprouts})$

[the dual]

- Not everyone dislikes Brussel sprouts.

Equality

$term_1 = term_2$ is true under a given interpretation
if and only if $term_1$ and $term_2$ refer to the same object

E.g., definition of (full) Sibling in terms of Parent

$$\forall x, y \text{ Sibling}(x,y) \Leftrightarrow [\neg(x=y) \wedge \exists m,f \neg(m=f) \wedge \\ \text{Parent}(m,x) \wedge \text{Parent}(f,x) \wedge \\ \text{Parent}(m,y) \wedge \text{Parent}(f,y)]$$

Fun with sentences

Brothers are siblings

$$\forall x, y \text{ Brother}(x,y) \Rightarrow \text{Sibling}(x,y)$$

"Sibling" is reflexive

$$\forall x, y \text{ Sibling}(x,y) \Leftrightarrow \text{Sibling}(y,x)$$

One's mother is one's female parent

$$\forall x, y \text{ Mother}(x,y) \Leftrightarrow (\text{Female}(x) \wedge \text{Parent}(x,y))$$

A first cousin is a child of a parent's sibling

$$\forall x, y \text{ FirstCousin}(x,y) \Leftrightarrow \exists p, ps \text{ Parent}(p,x) \wedge \text{Sibling}(ps,p) \wedge \text{Parent}(ps,y)$$

A tough one

- “Every city has a dogcatcher who has been bitten by every dog in town”
- Consider predicates
 - City(x)
 - Dog(x)
 - Dogcatcher(x,y)
 - LivesIn(x,y)
 - Bit(x,y)
- $\forall x \text{ City}(x) \Rightarrow \exists y \text{ Dogcatcher}(x,y) \wedge (\forall z (\text{Dog}(z) \wedge \text{LivesIn}(x,z) \Rightarrow \text{Bit}(y,z)))$

Higher Order Logic

- In first order logic, we can quantify over objects and then represent predicates and functions on objects.
- In a higher order logic, we can quantify over predicates and functions.
- For example:
 - “Two objects are equal iff all predicates applied to them are equivalent”
 - $\forall x,y (x=y) \Leftrightarrow (\forall p p(x) \Leftrightarrow p(y))$
 - Two functions are equal iff they have the same value for all arguments
 - $\forall f, g (f=g) \Leftrightarrow (\forall x f(x) = g(x))$
- These are not allowed in first order logic.
- While higher order logic is more expressive, we do not know how to reason effectively and undecidable