Game Playing

Introduction to Artificial Intelligence
CSE 150
Lecture 5
April 17, 2007
Tax Day!
CSP Summary

- CSPs are a special kind of search problem:
  - States defined by values of a fixed set of variables
  - Goal test defined by constraints on variable values
- Backtracking = depth-first search with
  1) fixed variable order
  2) only legal successors
  3) Stop search path when constraints are violated.
- Forward checking prevents assignments that guarantee a later failure
- Heuristics:
  - Variable selection: most constrained variable
  - Value selection: least constraining value
- Iterative min-conflicts is usually effective in practice
READING

- Finish Chapter 5, Read Chapter 6!
This lecture

• Introduction to Games
• Perfect play (Min-Max)
  – Tic-Tac-Toe
  – Checkers
  – Chess
  – Go
• Resource limits
• $\alpha$-$\beta$ pruning
• Games with chance
  – Backgammon
Types of Games

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Games vs. search problems

Similar to state-space search but:

1. There’s an *Opponent* whose moves must be considered

2. Because of the combinatorial explosion, we can’t search the entire tree:

   Have to *commit* to a move based on an *estimate* of its goodness.

Heuristic mechanisms we used to decide which node to open next will be used to decide which MOVE to make next.
Games vs. search problems

We will follow some of the history:

– algorithm for perfect play [Von Neumann, 1944]

– finite horizon, approximate evaluation [Zuse, 1945; Shannon, 1950; Samuel, 1952-57]

– pruning to reduce costs [McCarthy, 1956]
Deterministic Two-person Games

- Two players move in turn
- Each Player knows what the other has done and can do
- Either one of the players wins (and the other loses) or there is a draw.
Games as Search: Grundy’s Game
[Nilsson, Chapter 3]

• Initially a stack of pennies stands between two players
• Each player divides one of the current stacks into two unequal stacks.
• The game ends when every stack contains one or two pennies
• The first player who cannot play loses

MAX

MINNIE
Grundy’s Game: States

Can Minnie formulate a strategy to always win?

Max’s turn

Minnie’s turn

Max’s turn

Minnie’s turn

Max’s turn

Minnie’s turn

MINNIE Loses

MAX Loses

MINNIE Loses
When is there a winning strategy for Max?

- When it is MIN’s turn to play, a win must be obtainable for MAX from all the results of the move (sometimes called an AND node).

- When it is MAX’s turn, a win must be obtainable from at least one of the results of the move (sometimes called an OR node).
Minimax

- **Initial state:** Board position and whose turn it is.
- **Operators:** Legal moves that the player can make
- **Terminal test:** A test of the state to determine if game is over; set of states satisfying terminal test are terminal states.
- **Utility function:** numeric value for outcome of game (leaf nodes).
  - E.g. +1 win, -1 loss, 0 draw.
  - E.g. # of points scored if tallying points as in backgammon.

- Assumption: Opponent will always choose operator (move) to maximize own utility.
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**Minimax**

Perfect play for deterministic, perfect-information games

Max’s strategy: choose action with highest minimax value

→ best achievable payoff against best play by min.

Consider a 2-ply (two step) game:

Max wants largest outcome --- Minnie wants smallest
Minimax Example
Game tree for a fictitious 2-ply game

MAX

MIN

Leaf nodes: value of utility function

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Minimax Algorithm

Function MINIMAX-DECISION (game) returns an operator
For each op in OPERATORS[game] do
    Value[op] ← MINIMAX-VALUE(Apply(op, game), game)
End
Return the op with the highest Value[op]

Function MINIMAX-VALUE (state, game) returns a utility value
If TERMINAL-TEST[game](state) then
    return UTILITY[game](state)
else if MAX is to move in state then
    return the highest MINIMAX-VALUE of Successors[state]
else return the lowest MINIMAX-VALUE of Successors[state]
Properties of Minimax

• Complete: Yes, if tree is finite
• Optimal: Yes, against an optimal opponent. Otherwise??
• Time complexity: $O(b^m)$
• Space complexity: $O(bm)$

Note: For chess, $b \approx 35$, $m \approx 100$ for a “reasonable game.”

→ Solution is completely infeasible

Actually only board $10^{40}$ board positions, not $35^{100}$: Checking for previously visited states becomes important! (“transposition table”)
Resource limits

Suppose we have 100 seconds, explore $10^4$ nodes/second

$\rightarrow 10^6$ nodes per move

Standard approach:

• cutoff test
  
  e.g., depth limit

• evaluation function = estimated desirability of position
  
  (an approximation to the utility used in minimax).
Evaluation Function

For chess, typically linear weighted sum of features

\[ \text{Eval}(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s) \]

e.g., \( w_1 = 9 \) with
\[ f_1(s) = (\text{number of white queens}) - (\text{number of black queens}) \]

\( w_2 = 5 \) with
\[ f_2(s) = (\text{number of white rooks}) - (\text{number of black rooks}) \]
Digression: Exact Values Don't Matter

- Behavior is preserved under any monotonic transformation of Eval

- Only the order matters:
  - Payoff in deterministic games acts as an ordinal utility function
Cutting off search

- Replace \textit{MinimaxValue} with \textit{MinimaxCutoff}.
- \textit{MinimaxCutoff} is identical to \textit{MinimaxValue} except
  1. \texttt{Terminal?} is replaced by \texttt{Cutoff?}
  2. \texttt{Utility} is replaced by \texttt{Eval}

Does it work in practice?

\[ b^m = 10^6, \quad b = 35 \quad \Rightarrow \quad m = 4 \]

No: 4-ply lookahead is a hopeless chess player!
  4-ply human novice
  8-ply typical PC, human master
  12-ply Deep Blue, Kasparov
Another idea

• $\alpha - \beta$ Pruning
• Essential idea is to stop searching down a branch of tree when you can determine that it’s a dead end.
Pruning Example

α–β Pruning Example

MAX

MIN

3  12  8

2

14  5  2

=3  ≤3 =3

=2

≤14 ≤5

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The $\alpha$–$\beta$ Algorithm

Basically MINIMAX and keep track of $\alpha$, $\beta$ and prune

```
function Max-Value(state, game, $\alpha$, $\beta$) returns the minimax value of state
    inputs: state, current state in game
    game, game description
    $\alpha$, the best score for MAX along the path to state
    $\beta$, the best score for MIN along the path to state
    if Cutoff-Test(state) then return Eval(state)
    for each $s$ in Successors(state) do
        $\alpha \leftarrow \max(\alpha, \text{Min-Value}(s, game, \alpha, \beta))$
        if $\alpha \geq \beta$ then return $\beta$
    end
    return $\alpha$

function Min-Value(state, game, $\alpha$, $\beta$) returns the minimax value of state
    if Cutoff-Test(state) then return Eval(state)
    for each $s$ in Successors(state) do
        $\beta \leftarrow \min(\beta, \text{Max-Value}(s, game, \alpha, \beta))$
        if $\beta \leq \alpha$ then return $\alpha$
    end
    return $\beta$
```
How is the search tree pruned?

- $\alpha$ is the best value (to MAX) found so far.
- If $V \leq \alpha$, then the subtree containing $V$ will have utility no greater than $V$
  \[ \rightarrow \text{Prune that branch} \]
- $\beta$ can be defined similarly from MIN’s perspective
- Note: The book’s figure 6.5 makes little sense to me w.r.t. the algorithm in 6.7
Properties of $\alpha-\beta$

Pruning does not affect final result.

Good move ordering improves effectiveness of pruning.

With ``perfect ordering,'' time complexity = $O(b^{m/2})$
  – doubles depth of search
  – can easily reach depth 8 and play good chess

A simple example of the value of reasoning about which computations are relevant (a form of metareasoning).
Deeper searches

The **horizon** problem:

Sometimes disastser (or Shangri-la) is just beyond the depth bound.

One fix is **quiescence** search:
If a piece is about to be taken, the board is not “quiet” - keep searching beyond that point. Deep blue used this.

Another (new) and useful heuristic:

the **null move** heuristic: Let the opponent take **two** moves and then search under the node.
Stored knowledge

- There is a big fan-out at the beginning of the game:
  - Deep blue stored thousands of opening moves
  - And thousands of *solved* end games (all games with five pieces or less)
  - And 700,000 grandmaster games
  - Nowadays, a PC with good heuristics is as good as the best player.
Deterministic games in practice

**Checkers:** In 1994 *Chinook* ended 40-year-reign of human world champion Marion Tinsley. Used an endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 443,748,401,247 positions.

**Chess:** *Deep Blue* defeated human world champion Gary Kasparov in a six-game match in 1997. Deep Blue searches 200 million positions per second, uses very sophisticated evaluation, and undisclosed methods for extending some lines of search up to 40 ply.

**Othello:** Human champions refuse to compete against computers, who are too good.

**Go:** Human champions refuse to compete against computers, who are too bad. In Go, \( b > 300 \), so most programs use pattern knowledge bases to suggest plausible moves.
Nondeterministic games

- E.g., in backgammon, the dice rolls determine the legal moves
- Consider simpler example with coin-flipping instead of dice-rolling:

```
MAX

CHANCE

MIN
```

![Game tree diagram with chance nodes and utility values](image)
Nondeterministic games

1. For mind node, compute min of children

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Nondeterministic games

1. For mind node, compute min of children
2. For chance node, compute weighted average of children
Nondeterministic games

1. For mind node, compute min of children
2. For chance node, compute weighted average of children
3. For max node, compute max of children

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Algorithm for Nondeterministic Games

**Expectiminimax** maximize expected value (average) of utility against a perfect opponent.

Pseudocode is just like **Minimax**, except we must also handle chance nodes:

...  
**if** state **is** a chance node **then**  
**return** average of  
ExpectiMinimax-Value of Successors(state)  
...  

A version of $\alpha-\beta$ pruning is possible but only if the leaf values are bounded.
Function MINIMAX-VALUE (state, game) returns a utility value
If TERMINAL-TEST[game](state) then
    return UTILITY[game](state)
else if MAX is to move in state then
    return the highest MINIMAX-VALUE of Successors[state]
else return the lowest MINIMAX-VALUE of Successors[state]

Function EXPECTIMINIMAX-VALUE (state, game) returns a utility value
If TERMINAL-TEST[game](state) then
    return UTILITY[game](state)
else if state is a chance node then
    return average of EXPECTIMINIMAX-VALUE of Successors[state]
else if MAX is to move in state then
    return the highest EXPECTIMINIMAX-VALUE of Successors[state]
else return the lowest EXPECTIMINIMAX-VALUE of Successors[state]
Nondeterministic Games in Practice

- Dice rolls increase branching factor $b$:
  - 21 possible rolls with 2 dice
  - Backgammon 20 legal moves

- $\text{depth} 4 \to 20 \times (21 \times 20)^3 \approx 1.2 \times 10^9$

- As depth increases, probability of reaching a given node shrinks
  $\to$ value of look ahead is diminished

- $\alpha-\beta$ pruning is much less effective

- TDGammon uses depth-2 search + very good Eval
  Plays at world-champion level
Digression: Exact Values DO Matter

- Behavior is preserved only by positive linear transformation of Eval.

- Hence Eval should be proportional to the expected payoff.
Games Summary

- Games are fun to work on! (and dangerous)

- They illustrate several important points about AI
  - perfection is unattainable $\Rightarrow$ must approximate
  - good idea to think about what to think about
  - uncertainty constrains the assignment of values to states

- “Games are to AI as grand prix racing is to automobile design”