(I) (Laws and Theorems of Boolean Algebra) Prove using Boolean algebra that
\[ a'c' + ab + ac + a'b' = a'c' + ab + b'c. \]
Write the particular law you are using in each step.

(II) (Laws and Theorems of Boolean Algebra) Prove using Boolean algebra that
\[ (a + c)(a' + c')(b' + c + d')(a + b' + d') = (a + c)(a' + c')(b' + d'). \]
Write the particular law you are using in each step.

(III) (Karnaugh Map) Use Karnaugh map to simplify function
\[ f(a, b, c, d) = \sum m(0,1,2,3,4,5,7,8,12) + \sum d(10,11). \]
List all possible minimal two-level sum of products expressions. Show the switching functions. No need for the diagram.

(IV) (Quine-McCluskey Method) Use the Quine-McCluskey method to find the minimum sum-of-products expression of function
\[ f(a, b, c, d, e, f) = \sum m(0,4,17,48) + \sum d(21,36,49). \]
Show the implication chart, and give the result in Boolean expressions.

(V) Universal Set of Gates: Check if the set in the following list is universal and explain your decision. Assuming constants 0 and 1 are available as inputs.

i. \{AND, NOT\}

ii. \{NAND\}

iii. \{XOR, NOT\}

iv. \{f(x, y)\}, where \( f(x, y) = x'y \)

v. \{g(x, y, z)\}, where \( g(x, y, z) = (x + y)z' \)

vi. \{f(x, y), g(x, y)\}, where \( f(x, y) = x'y + xy' \) and \( g(x, y) = x'y' \)

(VI) Flip-Flops: Construct a T flip-flop from a JK flip-flop. Show the logic diagram.