CSE252B – Computer Vision II – Final Exam
Instructor: Prof. Serge Belongie.
`http://www-cse.ucsd.edu/classes/sp06/cse252b`
3:00pm-6:00pm Wed. June 14, 2006.

On this exam you are allowed to use a calculator and two 8.5" by 11" sheets of notes. The total number of points possible is 40. In order to get full credit you must show all your work. Good luck!

1. (3 pts) Provide three different definitions of an epipole.
2. (3 pts) Show that an affine transformation can map a circle to an ellipse, but cannot map an ellipse to a hyperbola or parabola.
3. (1 pt) Derive the expression for C', the image of a conic C under a homography H.
4. Many natural and man-made objects (e.g., airplanes) exhibit bilateral symmetry. Suppose you capture a *single* image $I(x, y)$ of an airplane using a camera with unknown relative pose $(R, \boldsymbol{T})$ with respect to the coordinate frame of the airplane.
   (a) (4 pts) Given only $I(x, y)$ as input, explain how to estimate the 3D structure of the airplane using techniques from this course. This will only be possible up to a certain unknown transformation; name the class of that transformation.
   (b) (2 pts) The quality of the 3D reconstruction will depend on the relative camera pose. What are the worst choices of $(R, \boldsymbol{T})$? What are the best?
5. Consider the homogeneous transformation $H \in GL(4)$.
   (a) (1 pt) How many degrees of freedom does H have?
   (b) (1 pt) Given a set of points in $\mathbb{P}^3$, what does H represent?
   (c) (2 pts) How many ground truth coordinates are needed to estimate H? What condition must these points satisfy?
6. Suppose you capture two images related by a pure translation in the Z direction, e.g., forward motion through a hallway.
   (a) (1 pt.) Where is the epipole in this case?
   (b) (1 pt.) What problem arises if you try to apply standard epipolar rectification to this pair of images?
   (c) (1 pt.) Suggest a high-level approach to address this problem.
7. Consider a conic with coefficient matrix

$$C = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

   (a) (1 pt.) What kind of curve is this?
   (b) (2 pt.) Compute its intersection with the line at infinity.
8. (3 pts) What is the normal vector of the plane at infinity? What is the motivation for identifying its image under perspective projection? Show how to estimate the image of the plane at infinity by solving a null space problem.

9. Consider an image of the chalkboard in Center Hall 205 captured by a student sitting in class. Let $\boldsymbol{x}_1 \in \mathbb{P}^2$ denote the homogeneous coordinates of a point on the chalkboard, and let $\boldsymbol{x}_2 \in \mathbb{P}^2$ denote its image.

   (a) (1 pt) What general class of transformation $\mathcal{T}(\cdot)$ maps $\boldsymbol{x}_1$ to $\boldsymbol{x}_2$?

   (b) (2 pts) How many corresponding point pairs are needed to estimate $\mathcal{T}(\cdot)$? What are the conditions on the coordinates of these points for the solution to be valid?

   (c) (1 pt) Suppose the student is seated very far away from the chalkboard. What simplified class of global transformation can be used to approximate $\mathcal{T}(\cdot)$?

   (d) (2 pts) Now suppose the student is seated at an arbitrary location. The professor marks a point $\boldsymbol{x}_1^*$ on the chalkboard. Describe how you would compute a linear approximation $\widetilde{\mathcal{T}}(\cdot)$ to $\mathcal{T}(\cdot)$ that holds in the vicinity of $\boldsymbol{x}_1^*$. What class of transformation is $\widetilde{\mathcal{T}}(\cdot)$?

   (e) (3 bonus pts) Solve for $\widetilde{\mathcal{T}}(\cdot)$ in terms of the coefficients of $\mathcal{T}(\cdot)$.

10. (3 pts) After running the eight point algorithm to estimate F, what is the reason for setting its third singular value to zero? Although this zeroing operation helps in one sense, it hurts in another. In what respect is the zeroing operation a bad thing? What can you do in practice to fix this problem?

11. (2 pts.) Referring to the expression for a 3D point imaged by a general camera, i.e., $\lambda \boldsymbol{x} = \Pi \boldsymbol{X}$, with $\Pi = KR[I, \boldsymbol{T}]$, explain why distant points (e.g., on the moon or a mountain) appear stationary when viewed from a translating vehicle.

12. (3 pts) How are the interest point detection methods of Förstner and Harris similar? How do they differ?