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Day 10

Probability

Instructor: Neil Rhodes
Definitions

A sample space, $S$, is a set of elementary events
- Rolls of a die = $\{1, 2, 3, 4, 5, 6\}$
- Flipping two coins = $\{HH, HT, TH, HH\}$

An event is a subset of a sample space:
- Rolling better than a 4 = $\{5, 6\}$
- Flipping at least one head = $\{HH, HT, TH\}$

A probability distribution (or probability function), $P$, is a function which assigns a real value to each elementary event $t$ in $S$ such that:
- $P(t) \geq 0$
- The sum over all $t$ of $P(t)$ equals 1

- For an event $E$, $P(E) =$ the sum of the probabilities of each elementary event in $E$
  - Probability of rolling better than a 4 is $P(5) + P(6) = 2/6$
  - Probability of flipping at least one head is $P(HH) + P(HT) + P(TH) = 3/4$

A probability space is a combination of a sample space and a probability distribution
If the sample space $S$ is finite and for each elementary event $e$, $P(e) = 1/|S|$, then $P$ is a uniform probability distribution
- Probability distribution for rolling dice $\{1, 2, 3, 4, 5, 6\}$ is uniform
- Probability distribution for scores on the SAT $\{400, 410, \ldots 1590, 1600\}$ is not uniform
Theorem of Union of Events

If \((S, P)\) is a probability space and \(A\) and \(B\) are disjoint subsets of \(S\), then:

- \(P(A \cup B) = P(A) + P(B)\)

- For rolling dice, if \(A = \{5, 6\}\) and \(B = \{1, 2\}\), then \(P(\{1, 2, 5, 6\}) = \frac{2}{3}\)

In general, (\(A\) and \(B\) may or may not be disjoint):

- \(P(A \cup B) = P(A) + P(B) - P(A \cap B)\)

- For rolling dice, if \(A = \{5, 6\}\) and \(B = \{2, 4, 6\}\), then
  \[
P(\{5, 6\} \cup \{2, 4, 6\}) = P(\{5, 6\}) + P(\{2, 4, 6\}) - P(\{5, 6\} \cap \{2, 4, 6\})
  = \frac{1}{3} + \frac{1}{2} - P(\{6\})
  = \frac{1}{3} + \frac{1}{2} - \frac{1}{6}
  = \frac{2}{3}
\]
Example (from CL exercise 4.1)

Six horses are in a race. You pick two of them at random and bet on them both. Find the probability that one of your horses came in first.
Example (from CL exercise 4.4)

Six horses are in a race. You pick two of them at random and bet on them both. Find the probability that one (or two) of your horses came in first or second.
Example (from CL exercise 4.8)

An urn contains ten balls labeled 1..10.

Two balls are drawn together.
- What is the sample space?
- What is the probability that the sum is odd?

Two balls are drawn one after the other without replacement.
- What is the sample space?
- What is the probability that the sum is odd?

Two balls are drawn one after the other with replacement.
- What is the sample space?
- What is the probability that the sum is odd?
Example (from CL example 29)

Six light bulbs are chosen at random from 18 light bulbs of which 8 are defective. What is the probability that exactly two of the chosen bulbs are defective?

- \( B = \) total number of balls
- \( D = \) total number of defective balls
- \( b = \) number of balls chosen
- Probability space = set of ways to choose \( b \) bulbs from \( B \) with uniform distribution
- \( E(B, D, b, d) = \) event with all selections of \( b \) from \( B \) where \( D \) bulbs are defective and \( d \) of the selected bulbs are defective

- Want \( P(E(B, D, b, d)) \) (called the hypergeometric probability distribution)
Example

A box has 2 white socks and 2 blue socks. Two socks are drawn at random. Find the probability $p$ they are a match.