0. Label the starting vertex $V_0$ as 0.

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Given a directed graph $G = (V, E)$ where there is a length $d_{ij}$ on each arc (directed edge) from vertex $i$ to vertex $j$, the assumptions are:

1. $d_{ij} > 0$ for all $i, j$
2. $d_{ij} \neq d_{ji}$ for some $i, j$
3. $d_{ij} + d_{jk} \leq d_{ij}$ for all $i, j, k$
Properties of paths:

1. The length of a path is at least the length of its subpaths:
   \[ |\text{Path}| \geq |\text{Subpath}|. \]

2. Given a shortest path between two vertices, any subpath is also a shortest path.

3. Let \( P_k \) be the number of arcs (directed edges) from the starting vertex \( V_0 \) to its \( k^{\text{th}} \) nearest vertex. Then \( P_k \) is at most \( k \).
Dijkstra’s Algorithm 3

The algorithm:

0. Vertex $V_0$ is initialized with label $l_0^* = 0$.
   For all other vertices $V_i$:
   \[ l_i = d_{0,i} \quad \text{if } V_i \text{ is a neighbor of } V_0 \]
   \[ l_i = \infty \quad \text{otherwise} \]

1. Pick $l_k = \min_i l_i$, then update $l_k \leftarrow l_k^*$.

2. Relax vertex $k$’s neighbors:
   \[ l_i \leftarrow \min[l_i, l_k^* + d_{k,i}] \].