Announcements

- Assignment 0: Posted to web page, due on Thursday
- Matlab will be discussed during discussion section, Wed 9:00 - 9:50 AM in WLH 2208
- Read Trucco & Verri: pp. 15-40

Image Formation: Outline

- Factors in producing images
- Projection
- Perspective
- Vanishing points
- Orthographic
- Lenses
- Sensors
- Quantization/Resolution
- Illumination
- Reflectance

Earliest Surviving Photograph

- First photograph on record, “la table service” by Nicephore Niepce in 1822.
- Note: First photograph by Niepce was in 1816.

How Cameras Produce Images

- Basic process:
  - photons hit a detector
  - the detector becomes charged
  - the charge is read out as brightness

- Sensor types:
  - CCD (charge-coupled device)
    - high sensitivity
    - high power
    - cannot be individually addressed
    - blooming
  - CMOS
    - most common
    - simple to fabricate (cheap)
    - lower sensitivity, lower power
    - can be individually addressed
**Effect of Lighting: Monet**

**Change of Viewpoint: Monet**

**Jetty at Margate England, 1898.**

*Camera Obscura*

>。“When images of illuminated objects ... penetrate through a small hole into a very dark room ... you will see [on the opposite wall] these objects in their proper form and color, reduced in size ... in a reversed position, owing to the intersection of the rays”.

*Da Vinci*

*http://www.acmi.net.au/AIC/CAMERA_OBSCURA.html (Russell Naughton)*

**Pinhole Camera: **Perspective projection

- Abstract camera model - box with a small hole in it

- Used to observe eclipses (eg., Bacon, 1214-1294)
- By artists (eg., Vermeer).
Distant objects are smaller

(Forsyth & Ponce)

Geometric properties of projection

- Points go to points
- Lines go to lines
- Planes go to whole image or half-plane
- Polygons go to polygons
- Angles & distances not preserved

Degenerate cases:
- line through focal point yields point
- plane through focal point yields line

Parallel lines meet in the image

- vanishing point

Image plane

Vanishing points

To different directions correspond different vanishing points
The equation of projection

Cartesian coordinates:

- We have, by similar triangles, that \((x, y, z) \rightarrow (f x/z, f y/z, -f)\)
- Ignore the third coordinate, and get \((x, y, z) \rightarrow (f x/z, f y/z)\)

A Digression

Homogenous Coordinates

and

Camera Matrices

Homogenous coordinates

- Our usual coordinate system is called a Euclidean or affine coordinate system
- Rotations, translations and projection in Homogenous coordinates can be expressed linearly as matrix multiplies

Euclidean <-> Homogenous <-> Euclidean

In 2-D

- Euclidean -> Homogenous: \((x, y) \rightarrow k (x,y,1)\)
- Homogenous -> Euclidean: \((u,v,w) \rightarrow (u/w, v/w)\)

In 3-D

- Euclidean -> Homogenous: \((x, y, z) \rightarrow k (x,y,z,1)\)
- Homogenous -> Euclidean: \((x, y, z, w) \rightarrow (x/w, y/w, z/w)\)

The camera matrix

\[(x, y, z) \rightarrow (f x/z, f y/z)\]

Turn this expression into homogenous coordinates

- HC’s for 3D point are \((X,Y,Z,T)\)
- HC’s for point in image are \((U,V,W)\)

Perspective Camera Matrix

A 3x4 matrix
End of the Digression

Affine Camera Model

- Take Perspective projection equation, and perform Taylor Series Expansion about (some point \((x_0, y_0, z_0)\).
- Drop terms of higher order than linear.
- Resulting expression is affine camera model

\[
\begin{bmatrix}
    u \\
    v \\
    w
\end{bmatrix} = f \begin{bmatrix}
    x \\
    y \\
    z
\end{bmatrix}
\]

- Perspective
- Assume that \(f=1\), and perform a Taylor series expansion about \((x_0, y_0, z_0)\)
- Dropping higher order terms and regrouping.

\[
\begin{bmatrix}
    u \\
    v \\
    w
\end{bmatrix} = \begin{bmatrix}
    1/z_0 & 0 & -x_0/z_0 & 0 \\
    0 & 1/z_0 & -y_0/z_0 & 0 \\
    0 & 0 & 1 & 0
\end{bmatrix} \begin{bmatrix}
    x \\
    y \\
    z \\
    1
\end{bmatrix} = Ap + b
\]

Orthographic projection

Starting with Affine camera mode
Take Taylor series about \((0, 0, z_0)\) – a point on optical axis

\[
\begin{bmatrix}
    u \\
    v \\
    w
\end{bmatrix} = \begin{bmatrix}
    1/z_0 & 0 & 0 & 0 \\
    0 & 1/z_0 & 0 & 0 \\
    0 & 0 & 1 & 0
\end{bmatrix} \begin{bmatrix}
    x \\
    y \\
    z \\
    1
\end{bmatrix}
\]

Parallel lines project to parallel lines
Ratios of distances are preserved under orthographic

The projection matrix for orthographic projection
Other camera models

- Generalized camera – maps points lying on rays and maps them to points on the image plane.

Omnicam (hemispherical)  
Light Probe (spherical)

Some Alternative “Cameras”

What if camera coordinate system differs from object coordinate system

\[
\begin{align*}
\{c\} & \rightarrow \{W\} \\
\mathbf{P} & \rightarrow \mathbf{p}
\end{align*}
\]

Euclidean Coordinate Systems

\[
\begin{align*}
x &= \mathbf{OP}_x \\
y &= \mathbf{OP}_y \\
z &= \mathbf{OP}_z \\
\mathbf{P} &= \begin{bmatrix} x \\ y \\ z \end{bmatrix}
\end{align*}
\]

Coordinate Changes: Pure Translations

Not rotation (e.g., \(i_x = i_4\), etc)

\[
\mathbf{O_B P} = \mathbf{O_B O_A} + \mathbf{O_A P} \quad \mathbf{B P} = \mathbf{A P} + \mathbf{B O_A}
\]

Rotation Matrix

\[
\mathbf{A^R} = \begin{bmatrix}
\mathbf{i_A} & \mathbf{j_A} & \mathbf{k_A} \\
\mathbf{i_B} & \mathbf{j_B} & \mathbf{k_B} \\
\mathbf{i_4} & \mathbf{j_4} & \mathbf{k_4}
\end{bmatrix} = 
\begin{bmatrix}
\mathbf{i_A} & \mathbf{j_A} & \mathbf{k_A} \\
\mathbf{i_B} & \mathbf{j_B} & \mathbf{k_B} \\
\mathbf{i_4} & \mathbf{j_4} & \mathbf{k_4}
\end{bmatrix} = 
\begin{bmatrix}
\mathbf{i_A} & \mathbf{j_A} & \mathbf{k_A} \\
\mathbf{i_B} & \mathbf{j_B} & \mathbf{k_B} \\
\mathbf{i_4} & \mathbf{j_4} & \mathbf{k_4}
\end{bmatrix}
\]
A rotation matrix $R$ has the following properties:

- Its inverse is equal to its transpose $R^{-1} = R^T$.
- Its determinant is equal to 1: $\det(R) = 1$.

Or equivalently:

- Rows (or columns) of $R$ form a right-handed orthonormal coordinate system.

Coordinate Changes: Pure Rotations

\[
\overrightarrow{OP} = \begin{bmatrix} i_x & j_y & k_z \end{bmatrix} \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{bmatrix} = \begin{bmatrix} i_R & j_R & k_R \end{bmatrix} \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{bmatrix}
\]

$\Rightarrow \overrightarrow{P} = R \overrightarrow{P}$

Coordinate Changes: Rigid Transformations

\[
^B P = ^A R \ P^A + ^B O_A
\]