Homework #4 Solutions.

2.5

a.

b.

c.

d.
Three cases, more a’s than b’s, more b’s than a’s, and some a after some b are all in this language. These descriptions overlap, creating an ambiguous language.

\[ A \rightarrow B | D | F \]
\[ B \rightarrow C \ b \ C \ a \ C \]
\[ C \rightarrow aC \ | \ bC \ | \ \epsilon \]
\[ D \rightarrow E \ a \ E \]
\[ E \rightarrow EE \ | \ aE b \ | \ bE a \ | \ a \ | \ \epsilon \]
\[ F \rightarrow G \ b \ G \]
\[ G \rightarrow GG \ | \ aGb \ | \ bGa \ | \ b \ | \ \epsilon \]
d. Will come out later today.

2.7 b

From a single start state, run an epsilon transition to two options. The case for a “b” somewhere before an “a” is easily solved in classic FSA notation, looping through some number of a’s, then transitioning to another state on a b, and lastly an accepting state on receiving another “a”. The other “counting” case will push onto the stack an “a” if there isn’t a “b” on the top of the stack, in which case it will instead pop that “b”. Likewise, it will push a “b” on top of the stack if there isn’t an “a”, otherwise popping said “a” from the stack. The PDA will then test at the end of the input if the stack is not empty ($) and will accept.

2.7 d
Will also come out later today…

2.14

A $\rightarrow$ BAB | B | $\varepsilon$
B $\rightarrow$ 00 | $\varepsilon$

Add a new start state
S $\rightarrow$ A
A $\rightarrow$ BAB | B | $\varepsilon$
B $\rightarrow$ 00 | $\varepsilon$

Remove A $\rightarrow$ $\varepsilon$
S $\rightarrow$ A | $\varepsilon$
A $\rightarrow$ BAB | B
B $\rightarrow$ 00 | $\varepsilon$

Remove B $\rightarrow$ $\varepsilon$
S $\rightarrow$ A | $\varepsilon$
A $\rightarrow$ BAB | AB | BA | B
B $\rightarrow$ 00

Remove A$\rightarrow$ B
S $\rightarrow$ A | $\varepsilon$
A $\rightarrow$ BAB | AB | BA | 00
B $\rightarrow$ 00

Remove S$\rightarrow$ A
Convert to proper form
S $\rightarrow$ BAB | AB | BA | 00 | $\varepsilon$
A $\rightarrow$ BAB | AB | BA | 00
B $\rightarrow$ 00
Now, remove more than two non-terminals on RHS:
\[
S \rightarrow BC \mid AB \mid BA \mid 00 \mid e \\
A \rightarrow BC \mid AB \mid BA \mid 00 \\
B \rightarrow 00 \\
C \rightarrow AB
\]

Finally, remove more than one terminal on RHS:
\[
S \rightarrow BC \mid AB \mid BA \mid ZZ \mid e \\
A \rightarrow BC \mid AB \mid BA \mid ZZ \\
B \rightarrow ZZ \\
C \rightarrow AB \\
Z \rightarrow 0
\]

2.27
a.
b.

Stmt \(\rightarrow\) \(<\text{Assign}>\mid<\text{MIF}>\mid<\text{UIF}>

MIF \(\rightarrow\) if condition then \(<\text{MIF}>\) else \(<\text{MIF}>\) | Stmt

UIF \(\rightarrow\) if condition then \(<\text{Stmt}>\) | if condition then \(<\text{MIF}>\) else \(<\text{UIF}>\)

Assign \(\rightarrow\) \(A \ :=\ 1\)