Homework #2 Solutions.

1.16
a.

b.

1.18
a. $1\Sigma^*0$
b. $\Sigma^*1\Sigma^*1\Sigma^*1\Sigma^*$
c. $\Sigma^*0101\Sigma^*$
d. $\Sigma\Sigma0\Sigma^*$
e. $(0(\Sigma\Sigma)^*)\cup(1\Sigma(\Sigma\Sigma)^*)$
f. $(0\cup10)^*1^*$
g. $\Sigma\Sigma\Sigma\Sigma\cup\Sigma\Sigma\Sigma\Sigma\cup\Sigma\Sigma\Sigma\cup\Sigma\Sigma\cup\Sigma\cup\varepsilon$
h. $\varepsilon\cup1\cup0\cup1(0\cup11(0\cup1))(0\cup1)^*$
i. $(1\Sigma)^*$
j. $000^*\cup(000^*1\cup100\cup010)0^*$
k. $\varepsilon\cup0$
l. $(1^*01^*01^*)^*$
m. Ø
n. $(0 \cup 1)(0 \cup 1)^*$

1.19

a.

![Diagram](image1)

b.

![Diagram](image2)

c.

![Diagram](image3)

1.20

c.
In: aaab
In: b
Not in: aba
Not in: bbbbbbbba

d.
In: aaa
In: aaaaaa
Not in: b
Not in: aaaaa

e.
In: aaaaabbbbaaaaa
In: aaba
Not in: aaaaaabb
Not in: bab

f.
In: aba
In: bab
Not in: abab
Not in: bb

1.21

a.

b.
a. text, page 96

b. Let \( w = a^p b^p \). Then \( s = a^p b^p a b^p \). Now, with \( |xy| \leq p \), then \( y \) must either be an \( a \) with some \( b \)'s, or all \( b \)'s. In the former case, pumping up will result in multiple \( a \)'s in the first \( w \) (of \( www \)) that are not repeated in the remaining two instances. Likewise, in the latter case, where \( y \) contains strictly \( b \)'s, then pumping up will result in too many \( b \)'s for the first instance that are, again, not echoed in the remaining instances of the substring.

c. text, page 96
a. To show $a$ is not pumpable, let $s$ be the string $0^p10^p$. By $|xy| \leq p$, $y$ must contain only 0s. Let $y$ be $0^k$, where $0 \leq k \leq p$. Now, if we pump up the string $s$, we get $0^{p+k}10^p$ for each pump $i$, which is not in the accepted language.

b. text, page 97

c. Let $s$ be the string $0^p10^p$. By $|xy| \leq p$, $y$ must contain only 0s. Pumping up will result in a string that is no longer a palindrome. Since regular languages are close under complement, if non-palindromes were regular, then $L$ would be too.

d. Let $s$ be the string $0^p110^p1$. By $|xy| \leq p$, $y$ must contain only 0s. Let $y$ be $0^k$, where $0 \leq k \leq p$. When we pump on this $y$, we will end up with $0^{p+k}110^p1$, which is not in the accepted language.

1.49

Let $s$ be the string $0^p10^p$. By $|xy| \leq p$, $y$ must contain only 0s. If we pump down in this case, then the resulting string will be $0^{p-k}10^p$, which is not in the accepted language.

1.55

a. 4
b. 1
c. 1
d. 3
e. 2