CSE 105—Theory of Computability

Lecture 9—May 2, 2006
Pushdown Automata/CFG
Pumping Lemma for CFL

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Example

Language = \textit{w not} a palindrome (over \{0, 1\})
Example

Language: $L\{xy \mid x \neq y, x, y \in \{0, 1\}^*\}$
CFG can be converted to NPDA

Easiest to use Chomsky Normal Form
- Book uses any grammar without conversion; idea is the same

Let’s look at leftmost derivation with CNF
- Example grammar
  - \( S \rightarrow TT \mid RT \)
  - \( T \rightarrow 0 \mid TT \mid 1 \)
  - \( R \rightarrow 0 \mid RR \)

- Example string
  - 01101
CFG can be converted to NPDA

**Without Chomsky Normal Form**

- Can have arbitrary mix of terminals and non-terminals in sentential form
- Store everything except leading terminals on stack
- Match input symbols to stack terminal symbols
- Example:
  - S→0T1 | 1
  - T→T0 | ε
- Example string:
  - 0001
Still need to show can convert PDA to CFG
- Believe me:) (In book, if you desire)
Pumping Lemma

A CFL can be pumped up (any string over a certain length can be used as a pattern for infinitely many more strings)
Pumping Lemma

For every Context-Free language, $L$

- There exists $p$
- such that, for all strings $s \in L$, with $|s| \geq p$
- $s$ can be rewritten as $uvwxy$ with $|vwx| \leq p$, $|vx| \geq 0$
- where, for all $i \geq 0$, $uv^iwx^iy \in L$
Example

$L = \{ww | w \in \mathbb{0}^*\}$
Example

$L = \{w_1w | w \epsilon 0^*\}$
Palindromes
Example

\[ L = \{0^n1^n2^n \mid n \geq 0\} \]
Example

$L = \{ww | w \in \{0,1\}^*\}$
Example

CFL not closed under intersection