In Max Flows

There is a source $s$ and a sink $t$

In all other nodes, the flow is balanced.
\[ F(s, t) = C(X, \overline{X}) \]

\[ s \in X \quad t \in \overline{X} \]

Max Flow Min Cut Theorem
\[ F(s, t) = 2 \]
In Multi-Terminal Flows

We pick one pair of nodes as $s$ & $t$ at one time

$$F(g, h) = 2 \quad F(e, f) = 3$$
Also \( b_{i,j} = b_{j,i} \)

i.e. \( F(p, q) = F(q, p) \)

\[
\begin{align*}
F(a, b) &= F(b, a) = 7 \\
F(b, c) &= F(c, b) = 7 \\
F(a, c) &= F(c, a) = 8
\end{align*}
\]
\[ F(a, b) = F(b, a) = 7 \]
\[ F(b, c) = F(c, b) = 7 \]
\[ F(a, c) = F(c, a) = 8 \]
Given a $n$–node network

$$F(i, j), \quad F(j, k), \quad F(i, k)$$

Satisfy

$$F(i, k) \geq \min [F(i, j), F(j, k)]$$
$$F(i, j) \geq \min [F(i, k), F(j, k)]$$
$$F(j, k) \geq \min [F(i, j), F(i, k)]$$
\[ F(i, k) = C(X, \overline{X}) \]

Case I  
\( j \in X \quad F(j, k) \leq C(X, \overline{X}) \)

Case II  
\( j \in \overline{X} \quad F(i, j) \leq C(X, \overline{X}) \)

\[ F(i, k) \geq \min \{ F(i, j), F(j, k) \} \]
Among the three max flow values, two values must be the same, the third is ... ?

\[ F(i, k) = 2, \quad F(i, j) = 5, \quad F(j, k) = 2 \]
Max Flow Values

```
      a
    /   \
   /     \
  j ----- a ----- k
    |     |     |     |
    |     |     |     |
    |     b ----- i
    |     |
    |     |
    v     v
i

```
Assuming that we have the complete graph, where every link represents a Max Flow value.

Construct the MAX spanning tree.

Then we have a tree-shaped network which is flow equivalent.