Algorithm for Max Flow

- Based on the current flow pattern, locate a path from $s$ to $t$.
- Send flow along this path.
Max Flow

\begin{figure}
\centering
\begin{tikzpicture}[node distance = 2cm, line width=0.5mm]
\node (s) at (0,0) {$s$};
\node (i) at (2,0) {$i$};
\node (j) at (4,0) {$j$};
\node (k) at (4,-2) {$k$};
\node (t) at (6,0) {$t$};
\node (d) at (2,2) {$d$};
\node (3) at (1,1) {$3$};
\node (3) at (3,1) {$3$};
\node (1) at (2,0) {$1$};
\node (2) at (4,0) {$2$};
\node (2) at (4,-1) {$2$};
\node (3, s^+) at (1,3) {$3, s^+$};
\node (3, s^+) at (3,3) {$3, s^+$};
\node (1, i^+) at (3,1) {$1, i^+$};
\node (1, i^+) at (5,1) {$1, j^+$};
\node (1, i^+) at (2,-1) {$1, i^+$};
\draw[->, thick] (s) -- (i) node[midway, above] {3};
\draw[->, thick] (i) -- (j) node[midway, above] {1};
\draw[->, thick] (j) -- (t) node[midway, above] {2};
\draw[->, thick] (i) -- (d) node[midway, above] {1};
\draw[->, thick] (d) -- (s) node[midway, above] {3};
\draw[->, thick] (i) -- (k) node[midway, above] {1};
\draw[->, thick] (k) -- (d) node[midway, above] {1};
\draw[->, thick] (j) -- (k) node[midway, above] {2};
\draw[->, thick] (d) -- (t) node[midway, above] {3};
\draw[->, thick] (k) -- (i) node[midway, above] {1};
\end{tikzpicture}
\end{figure}
Step 1. Labeling Process

(i) Unlabeled
(ii) Labeled and Unscanned
(iii) Labeled and Scanned
Max Flow

\[ x_{i,j} < b_{i,j} \]

\[ \epsilon(i), s^+ \quad \epsilon(j), i^+ \]

\[ i \quad j \]

\[ \epsilon(i) = \text{amount at } v_i \]

\[ \epsilon(j) = \min[\epsilon(i), b_{i,j} - x_{i,j}] \]
Max Flow

\[ x_{j,i} > 0 \]

\[ \epsilon(i), s^+ \quad \epsilon(j), i^- \]

\[ \epsilon(j) = \min[\epsilon(i), x_{j,i}] \]
Max Flow

\begin{align*}
(j, i^-) &: (1, 1) \\
(j, j^+) &: (1, 0) \\
(j, s^+) &: (1, 0) \\
(s, j) &: (1, 1) \\
(s, i) &: (1, 1) \\
(i, j) &: (1, 0) \\
(i, t) &: (1, 1) \\
(t, j) &: (1, 1) \\
(t, s) &: (1, 1)
\end{align*}
Label the nodes given the current $x_{i,j}$ given here.