CSE 202: Homework 3 (Group)

Due: Day 16

When presenting an algorithm, make sure you have:

- A clear (pseudo-code) description of the algorithm
- A proof that it is correct (loop invariants may help)
- An analysis of its runtime (stating the runtime isn’t sufficient; provide a proof or justification).

For full credit, you need all three pieces.

Exercises

1. CLRS, exercise 14.3-6
2. CLRS, exercise 17-2
3. Given a string \( x \) consisting of 0s and 1s, we write \( x^k \) to denote \( k \) copies of \( x \) concatenated together. We say that a string \( x' \) is a repetition of \( x \) if it is a prefix of \( x^k \) for some number \( k \). So \( x' = 10110110110 \) is a repetition of \( x = 101 \).

We say that a string \( s \) is an interleaving of \( x \) and \( y \) if its symbols can be partitioned into two (not necessarily contiguous) subsequences \( s' \) and \( s'' \), so that \( s' \) is a repetition of \( x \) and \( s'' \) is a repetition of \( y \). (So each symbol in \( s \) must belong to exactly one of \( s' \) or \( s'' \).) For example, if \( x = 101 \) and \( y = 00 \), then \( s = 100010101 \) is an interleaving of \( x \) and \( y \), since characters 1, 2, 5, 7, 8, 9 form 101101--a repetition of \( x \)-- and the remaining characters 3, 4, 6 form 000--a repetition of \( y \).

Give an efficient algorithm that takes strings \( s, x, \) and \( y \) and decides if \( s \) is an interleaving of \( x \) and \( y \).
4. The owners of an independently operated gas station are faced with the following situation. They have a large underground tank in which they store gas; the tank can hold up to $L$ gallons at one time. Ordering gas is quite expensive, so they want to order relatively rarely. For each order, they need to pay a fixed price $P$ for delivery in addition to the cost of the gas ordered. However, it costs $c$ to store a gallon of gas for an extra day, so ordering too much ahead increases the storage cost.

They are planning to close for a week in the winter, and they want their tank to be empty by the time they close. Luckily, based on years of experience, they have accurate projections for how much gas they will need each day until this point in time. Assume that there are $n$ days left until they close, and they need $g_i$ gallons of gas for each of the days $i = 1, \ldots, n$. Assume that the tank is empty at the end of day 0. Give an algorithm to decide on which days they should place orders, and how much to order so as to minimize their total cost.