Image Formation and Cameras

Introduction to Computer Vision
CSE 152
Lecture 4

Announcements
• Assignment 0: due today
• Assignment 1: Will be posted prior to next class – mixture of very short programming assignments + written assignments.
• Read Trucco & Verri: pp. 15-40

Pinhole Camera: Perspective projection
• Abstract camera model - box with a small hole in it

Geometric Aspects of Perspective Projection
• Points project to points
• Lines project to lines
• Angles & distances (or ratios) are NOT preserved under perspective
• Vanishing point

The equation of projection
Cartesian coordinates:
• We have, by similar triangles, that \((x, y, z) \rightarrow \left(\frac{fx}{z}, \frac{fy}{z}, -f\right)\)
• Ignore the third coordinate, and get \((x, y) \rightarrow (\frac{fx}{z}, \frac{fy}{z})\)

Euclidean -> Homogenous-> Euclidean
In 2-D
• Euclidean -> Homogenous: \((x, y) \rightarrow k (x,y,1)\) (can just take \(k=1\))
• Homogenous -> Euclidean: \((x, y, z) \rightarrow (x/z, y/z)\)

In 3-D
• Euclidean -> Homogenous: \((x, y, z) \rightarrow k (x,y,z,1)\) (can just take \(k=1\))
• Homogenous -> Euclidean: \((x, y, z, w) \rightarrow (x/w, y/w, z/w)\)
The camera matrix

Turn

\[ (x,y,z) \rightarrow \left( \frac{x}{z}, \frac{y}{z} \right) \]

into homogenous coordinates

– HC’s for 3D point are \((X,Y,Z,1)\)
– HC’s for point in image are \((U,V,W)\)

\[
\begin{pmatrix}
U \\
V \\
W
\end{pmatrix} =
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
X \\
Y \\
Z
\end{pmatrix}
\]

Affine Camera Model

• Take Perspective projection equation, and perform Taylor Series Expansion about (some point \((x_0,y_0,z_0)\).
• Drop terms of higher order than linear.
• Resulting expression is called affine camera model.
• Properties
  – Pts. map to pts, lines map to lines
  – Parallel lines map to parallel lines (no vanishing point – at infinity)
  – Ratios of distance/angles preserved

Orthographic projection

Start with affine camera model, and take Taylor series about \((x_0, y_0, z_0) = (0, 0, z_0)\) – a point on optical axis

\[ x' = x \]
\[ y' = y \]

Depth \((z)\) is lost

Coordinate Changes: Rigid Transformations

\[
^B P = ^A R ^A P + ^B O_A
\]

Coordinate Changes: Pure Rotations

\[
\mathbf{R} = \begin{bmatrix}
i_x & i_y & i_z \\
j_x & j_y & j_z \\
k_x & k_y & k_z
\end{bmatrix}
\]

\[
^A \mathbf{P} = \mathbf{R} ^A \mathbf{P}
\]

A rotation matrix \(R\) has the following properties:
• Its inverse is equal to its transpose \(R^T = R^{-1}\)
• Its determinant is equal to 1: \(\det(R) = 1\).
Or equivalently:
• Rows (or columns) of \(R\) form a right-handed orthonormal coordinate system.

Rotation Matrix

Pure Rotation

\[
^B \mathbf{R} = \begin{bmatrix}
i_x & i_y & i_z \\
j_x & j_y & j_z \\
k_x & k_y & k_z
\end{bmatrix}
\]

Or equivalently:

\[
\begin{pmatrix}
i_A \\
j_A \\
k_A
\end{pmatrix} = \begin{pmatrix}
i_B \\
j_B \\
k_B
\end{pmatrix}
\]

\[
\begin{pmatrix}
i_A \\
j_A \\
k_A
\end{pmatrix} = \begin{pmatrix}
i_A \\
j_A \\
k_A
\end{pmatrix}
\]
Rotation

- About \((k_x, k_y, k_z)\), a unit vector on an arbitrary axis (Rodrigues Formula)

\[
\begin{bmatrix}
  x' \\
y' \\
z'
\end{bmatrix} = \begin{bmatrix}
  k_x(k(1-c)+c) & k_x(k(1-c)+ks) & k_x(k(1-c)+ks) \\
  k_y(k(1-c)+ks) & k_y(k(1-c)+c) & k_y(k(1-c)+ks) \\
  k_z(k(1-c)-ks) & k_z(k(1-c)+c) & k_z(k(1-c)+c)
\end{bmatrix} \begin{bmatrix}
  x \\
y \\
z
\end{bmatrix}
\]

where \(c = \cos \theta\) \& \(s = \sin \theta\)

Block Matrix Multiplication

\[
A = \begin{bmatrix}
  A_{11} & A_{12} \\
  A_{21} & A_{22}
\end{bmatrix}, \quad B = \begin{bmatrix}
  B_{11} & B_{12} \\
  B_{21} & B_{22}
\end{bmatrix}
\]

What is \(AB\)?

\[
AB = \begin{bmatrix}
  A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\
  A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22}
\end{bmatrix}
\]

Homogeneous Representation of Rigid Transformations

\[
\begin{bmatrix}
  \theta p + M_0 \\
  \theta p + R^T O_p
\end{bmatrix} = \begin{bmatrix}
  \theta R & \theta O \\
  0 & 1
\end{bmatrix} \begin{bmatrix}
  p \\
  1
\end{bmatrix}
\]

Transformation represented by 4 by 4 Matrix

Camera parameters

- Issue
  - camera may not be at the origin, looking down the \(z\)-axis
  - one unit in camera coordinates may not be the same as one unit in world coordinates
    - extrinsic parameters - focal length, principal point, aspect ratio, angle between axes, etc.
    - intrinsic parameters

\[
\begin{bmatrix}
  T \\
  F
\end{bmatrix} = \begin{bmatrix}
  1 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0
\end{bmatrix}
\]

Getting more light – Bigger Aperture

Camera Calibration

- Given \(n\) points \(P_1, \ldots, P_n\) with known positions and their images \(p_1, \ldots, p_n\), estimate intrinsic and extrinsic camera parameters

\[
\text{Estimation}
\]

Limits for pinhole cameras

2.16. 

LIMITS THE QUALITY OF PINHOLE OPTICS. These three images of a stripe element were made using pinholes with decreasing size. (a) When the pinhole
is abnormally large, the image size is not properly converged, and the image is blurry. (b) Reducing the size of the pinhole improves the focus. (c) Further reducing the size of the pinhole further worsens the focus, due to diffraction, from Danchaivijitr, 1998.
Pinhole Camera Images with Variable Aperture

2 mm

.6 mm

.15 mm

1 mm

.35 mm

.07 mm

The reason for lenses

Thin Lens

• Rotationally symmetric about optical axis.
• Spherical interfaces.

Thin Lens: Center

• All rays that enter lens along line pointing at O emerge in same direction.

Thin Lens: Focus

Parallel lines pass through the focus, F

Thin Lens: Image of Point

All rays passing through lens and starting at P converge upon P'
Thin Lens: Image of Point

\[ \frac{1}{z'} - \frac{1}{z} = \frac{1}{f} \]

A price: Whereas the image of \( P \) is in focus, the image of \( Q \) isn’t.

Thin Lens: Image Plane

\[ \frac{1}{z'} - \frac{1}{z} = \frac{1}{f} \]

Thin Lens: Aperture

- Smaller Aperture -> Less Blur
- Pinhole -> No Blur

Field of View

Spherical aberration

rays parallel to the axis do not converge

outer portions of the lens yield smaller focal lengths

Deviations from the lens model

Deviations from this ideal are aberrations

Two types

1. geometrical
   - spherical aberration
   - astigmatism
   - distortion
   - coma

2. chromatic
   Aberrations are reduced by combining lenses

Compound lenses
Distortion
magnification/focal length different for different angles of inclination
Can be corrected! (if parameters are known)

Chromatic aberration
Index of refraction of lens depends on wavelength of light

Chromatic aberration
rays of different wavelengths focused in different planes
The image is blurred and appears colored at the fringe.
cannot be removed completely
sometimes achromatization is achieved for more than 2 wavelengths

Vignetting in Compound Lenses

Radiometry, Lighting, Intensity

Lighting
• Applied lighting can be represented as a function on the 4-D ray space (radiances)
• Special light sources
  – Point sources
  – Distant point sources
  – Strip sources
  – Area sources
• Common to think of lighting at infinity (a function on the sphere, a 2-D space)
Irradiance

- How much light is arriving at a surface?
- **Irradiance** -- power per unit area W/cm²
- Total power arriving at the surface is given by adding irradiance over all incoming angles

\[
I = \iiint E(x, y, \lambda, t) s(x, y) q(\lambda) dx dy d\lambda dt
\]

Camera’s sensor

- Measured pixel intensity is a function of irradiance integrated over
  - pixel’s area
  - over a range of wavelengths
  - For some time

Light at surfaces

Many effects when light strikes a surface -- could be:
- transmitted
- Skin, glass
- reflected
- mirror
- scattered
- milk
- travel along the surface and leave at some other point
- absorbed
- sweaty skin

Assume that
- surfaces don’t fluoresce -- e.g. scorpions, detergents
- surfaces don’t emit light (i.e. are cool)
- all the light leaving a point is due to that arriving at that point

BRDF

- **Bi-directional Reflectance Distribution Function**
  \[
p(\theta_{in}, \phi_{in} ; \theta_{out}, \phi_{out})
\]
- Function of
  - Incoming light direction: \(\theta_{in}, \phi_{in}\)
  - Outgoing light direction: \(\theta_{out}, \phi_{out}\)
- Ratio of incident irradiance to emitted radiance

Surface Reflectance Models

**Common Models**
- Lambertian
- Phong
- Physics-based
  - Specular
    [Blinn 1977], [Cook-Torrance 1982], [Ward 1992]
  - Diffuse
    [Hirashima, Kreuger 1993]
  - Generalized Lambertian
    [Oren, Nayar 1995]
  - Thoroughly Pitted Surfaces
    [Koenderink et al 1999]
- Phenomenological
  [Koenderink, Van Doom 1996]

**Arbitrary Reflectance**

- Non-parametric model
- Anisotropic
- Non-uniform over surface
- BRDF Measurement
  [Dana et al. 1999], [Marschner]

Lambertian Surface

At image location \((u, v)\), the intensity of a pixel \(x(u, v)\) is:

\[
x(u, v) = a(u, v) \ \hat{n}(u, v) \cdot [s \ \hat{s}]
\]

where
- \(a(u, v)\) is the albedo of the surface projecting to \((u, v)\).
- \(\hat{n}(u, v)\) is the direction of the surface normal.
- \(s\) is the light source intensity.