Image Formation and Cameras

Introduction to Computer Vision
CSE 152
Lecture 3

Announcements
• Assignment: posted to web page, due on Thursday
• Read Trucco & Verri: pp. 15-40

Class Time Change???
• Tuesday, Thursday 2:00-3:20???

Image Formation: Outline
• Factors in producing images
• Projection
• Perspective
• Vanishing points
• Orthographic
• Lenses
• Sensors
• Quantization/Resolution
• Illumination
• Reflectance

Earliest Surviving Photograph
• First photograph on record, “la table service” by Nicephore Niepce in 1822.
• Note: First photograph by Niepce was in 1816.

How Cameras Produce Images
• Basic process:
  – photons hit a detector
  – the detector becomes charged
  – the charge is read out as brightness
• Sensor types:
  – CCD (charge-coupled device)
    • high sensitivity
    • high power
    • cannot be individually addressed
    • blooming
  – CMOS
    • most common
    • simple to fabricate (cheap)
    • lower sensitivity, lower power
    • can be individually addressed
Images are two-dimensional patterns of brightness values. They are formed by the projection of 3D objects.

Effect of Lighting: Monet

Change of Viewpoint: Monet

Haystack at Chailly at sunrise (1865)

Pinhole Camera: Perspective projection

- Abstract camera model - box with a small hole in it

Camera Obscura

"When images of illuminated objects ... penetrate through a small hole into a very dark room ... you will see [on the opposite wall] these objects in their proper form and color, reduced in size ... in a reversed position, owing to the intersection of the rays".

Da Vinci

http://www.acmi.net.au/AIC/CAMERA_OBSCURA.html (Russell Naughton)

- Used to observe eclipses (eg., Bacon, 1214-1294)
- By artists (eg., Vermeer).
Jetty at Margate England, 1898.

http://brightbytes.com/cosite/collection2.html (Jack and Beverly Wilgus)

Distant objects are smaller

Geometric properties of projection
- Points go to points
- Lines go to lines
- Planes go to whole image or half-plane
- Polygons go to polygons
- Angles & distances not preserved
- Degenerate cases:
  - line through focal point yields point
  - plane through focal point yields line

Parallel lines meet in the image
- vanishing point

Vanishing points
To different directions correspond different vanishing points
Vanishing Points

The equation of projection

Cartesian coordinates:
• We have, by similar triangles, that
  \((x, y, z) \rightarrow \left(\frac{f}{x/z}, \frac{f}{y/z}, -f\right)\)
• Ignore the third coordinate, and get

A Digression

Homogenous Coordinates
and
Camera Matrices

Homogenous coordinates
A way to represent points in a projective space
1. Add an extra coordinate
e.g., \((x, y) \rightarrow (x, y, 1) = (u, v, w)\)
2. Impose equivalence relation
   such that \((\lambda \not= 0)\)
   \((u, v, w) \approx \lambda (u, v, w)\)
i.e., \((x, y, 1) \approx (\lambda x, \lambda y, \lambda)\)
3. “Point at infinity” – zero for last coordinate
e.g., \((x, y, 0)\)

Why do this?
• Possible to represent points “at infinity”
  • Where parallel lines intersect
  • Where parallel planes intersect
• Possible to write the action of a perspective camera as a matrix

Euclidean -> Homogenous-> Euclidean

In 2-D
• Euclidean -> Homogenous: \((x, y) \rightarrow k (x, y, 1)\)
• Homogenous -> Euclidean: \((x, y, z) \rightarrow (x/z, y/z)\)

In 3-D
• Euclidean -> Homogenous: \((x, y, z) \rightarrow k (x, y, z, 1)\)
• Homogenous -> Euclidean: \((x, y, z, w) \rightarrow (x/w, y/w, z/w)\)
The camera matrix

\[(x,y,z) \rightarrow \left( f \frac{x}{z}, f \frac{y}{z} \right)\]

Turn this expression into homogenous coordinates
- HC’s for 3D point are (X,Y,Z,T)
- HC’s for point in image are (U,V,W)

\[U = \frac{f_x}{z} X, \quad V = \frac{f_y}{z} Y, \quad W = 1\]

Perspective Camera Matrix
A 3x4 matrix

End of the Digression

Affine Camera Model

- Take Perspective projection equation, and perform Taylor Series Expansion about (some point \((x_0, y_0, z_0)\).
- Drop terms of higher order than linear.
- Resulting expression is affine camera model

\[\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{z_0} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \frac{1}{z_0} z_0 \begin{bmatrix} 1/2z_0^2 - x_0/z_0^2 \\ 1/2z_0^2 - y_0/z_0^2 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}\]

Rewrite Affine camera model in terms of Homogenous Coordinates

\[\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{z_0} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \frac{1}{z_0} z_0 \begin{bmatrix} x_0/z_0^2 \\ y_0/z_0^2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}\]

Orthographic projection

Starting with Affine camera mode
Take Taylor series about (0, 0, z_0) – a point on optical axis
The projection matrix for orthographic projection

\[
\begin{bmatrix}
U \\
V \\
W
\end{bmatrix} = \begin{bmatrix}
1/z_0 & 0 & 0 & 0 \\
0 & 1/z_0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
X \\
Y \\
Z \\
T
\end{bmatrix}
\]

Parallel lines project to parallel lines
Ratios of distances are preserved under orthographic projection

Other camera models

- Generalized camera – maps points lying on rays and maps them to points on the image plane.

Omnicam (hemispherical)  Light Probe (spherical)

Some Alternative “Cameras”

What if camera coordinate system differs from object coordinate system

Euclidean Coordinate Systems

\[
\begin{align*}
x &= \vec{OP} \cdot \hat{i} \\
y &= \vec{OP} \cdot \hat{j} \\
z &= \vec{OP} \cdot \hat{k}
\end{align*}
\]

\[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\]

Coordinate Changes: Pure Translations

\[
O_B^P = O_B^{O_A} + O_A^P, \quad B^P = A^P + B^{O_A}
\]
A rotation matrix $R$ has the following properties:

- Its inverse is equal to its transpose $R^{-1} = R^T$.
- Its determinant is equal to 1: $\det(R) = 1$.

Or equivalently:

- Rows (or columns) of $R$ form a right-handed orthonormal coordinate system.

Rotation: Homogenous Coordinates

- About z axis

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- About x axis:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- About y axis:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 & \sin \theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \theta & 0 & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$
Roll-Pitch-Yaw

$R = rot(i, \alpha)rot(j, \beta)rot(k, \varphi)$

Euler Angles

$R = rot(\hat{k}', \alpha)rot(\hat{j}', \beta)rot(\hat{k}, \varphi)$

Rotation

- About $(k_x, k_y, k_z)$, a unit vector on an arbitrary axis (Rodrigues Formula)

$$
\begin{pmatrix}
    x' \\
y' \\
z'
\end{pmatrix} =
\begin{pmatrix}
    k_x(1-c)+c & k_y(1-c)-cs & k_z(1-c)-ks \\
k_x(1-c)+ks & k_y(1-c)+c & k_z(1-c)+ks \\
0 & -k_z(1-c)+ks & k_y(1-c)+c
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
z
\end{pmatrix}
$$

where $c = \cos \theta$ & $s = \sin \theta$