Some counting
Consider $M$ images of $N$ points, how many unknowns?
1. Affix coordinate system to location of first camera location: $(M-1)*6$ Unknowns
2. 3-D Structure: $3*N$ Unknowns
3. Can only recover structure and motion up to scale. Why?

Total number of unknowns: $(M-1)*6 + 3N - 1$

The Eight-Point Algorithm (Longuet-Higgins, 1981)
Set $F_{33}$ to 1
Solve

Finding Feature points
(e.g. Harris & Stephenson’88; Shi & Tomasi’94)
Find points that differ as much as possible from all neighboring points

$SSD = \Delta^T M \Delta$

So, to detect corners
• Filter image.
• Compute the gradient everywhere.
• We construct $C$ over a window.
• Use linear algebra to find $\lambda_1$ and $\lambda_2$.
• If they are both big, we have a corner.

$C = \left[ \sum I_x^2, \sum I_x I_y, \sum I_y^2 \right]$
Feature matching

Evaluate normalized cross correlation (or sum of squared differences) for all features with similar coordinates
e.g. \((x', y') \in [x - \frac{m}{n}, x + \frac{m}{n}] \times [y - \frac{m}{n}, y + \frac{m}{n}]\)

Keep mutual best matches
Still many wrong matches!

St. Peter’s Panorama

Continuous Motion
• Consider a video camera moving continuously along a trajectory (rotating & translating).
• How do points in the image move
• What does that tell us about the 3-D motion & scene structure?

Reconstruction Results (Tomasi and Kanade, 1992)

Motion

Some problems of motion
1. Correspondence: Where have elements of the image moved between image frames
2. Reconstruction: Given correspondence, what is 3-D geometry of scene
3. Segmentation: What are regions of image corresponding to different moving objects
4. Tracking: Where have objects moved in the image? related to correspondence and segmentation.

Variations:
– Small motion (video),
– Wide-baseline (multi-view)
Motion

“When objects move at equal speed, those more remote seem to move more slowly.”
- Euclid, 300 BC

Simplest Idea for video processing

Image Differences

• Given image \( I(u,v,t) \) and \( I(u,v, t+\delta t) \), compute \( I(u,v, t+\delta t) - I(u,v,t) \).

• This is partial derivative: \( \frac{\partial I}{\partial t} \)

• At object boundaries, \( \frac{\partial I}{\partial t} \) is large and a is cue for segmentation

• Doesn’t tell which way stuff is moving

Background Subtraction

• Gather image \( I(x,y,t_0) \) of background without objects of interest (perhaps computed over average over many images).

• At time \( t \), pixels where \( |I(x,y,t)-I(x,y,t_0)| > \tau \) are labeled as coming from foreground objects

The Motion Field

Where in the image did a point move?

The Motion Field

What causes a motion field?

1. Camera moves (translates, rotates)
2. Object’s in scene move rigidly
3. Objects articulate (pliers, humans, animals)
4. Objects bend and deform (fish)
5. Blowing smoke, clouds
Motion Field Yields 3-D Motion Information
The "instantaneous" velocity of points in an image

The Focus of Expansion (FOE)
Intersection of velocity vector with image plane

With just this information it is possible to calculate:
1. Direction of motion
2. Time to collision

Is motion estimation inherent in humans?
Demo

Rigid Motion and the Motion Field

Rigid Motion: General Case
Position & Orientation

\[ \dot{p} = T + \omega \times p \]

General Motion

\[ \begin{bmatrix} \dot{u} \\ \dot{v} \end{bmatrix} = \frac{f}{z} \begin{bmatrix} x \\ y \end{bmatrix} \]

\[ \begin{bmatrix} \dot{u} \\ \dot{v} \end{bmatrix} = \frac{f}{z} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + \frac{\dot{z}}{z} \begin{bmatrix} x \\ y \end{bmatrix} \]

Substitute \( \dot{p} = T + \omega \times p \) where \( p=(x,y,z)^T \)

Motion Field Equation

\[ \dot{u} = \frac{T_u - T_f f - \omega_y f + \omega_x v + \omega_{uv} - \omega_x \mu^2}{Z} \]

\[ \dot{v} = \frac{T_v - T_f f + \omega_y f - \omega_x \mu - \omega_{uv} - \omega_y v^2}{Z} \]

- \( T \): Components of 3-D linear motion
- \( \omega \): Angular velocity vector
- \( (u,v) \): Image point coordinates
- \( Z \): depth
- \( f \): focal length
Pure Translation

\[ \begin{align*}
\dot{u} &= \frac{T_x u - T_y f}{Z} - \omega_y f + \omega_x v + \frac{\omega_u u v}{f} - \frac{\omega_u u^2}{f} \\
\dot{v} &= \frac{T_y v + T_x f}{Z} + \omega_y f - \omega_x v - \frac{\omega_u u v}{f} - \frac{\omega_v v^2}{f}
\end{align*} \]

\[ \omega = 0 \]

Forward Translation & Focus of Expansion

[Gibson, 1950]

Pure Translation

- Pure Rotation: \( T = 0 \)

- Independent of \( T_x, T_y, T_z \)
- Independent of \( Z \)
- Only function of \( (u,v), f \) and \( \omega \)

Rotational MOTION FIELD

The "instantaneous" velocity of points in an image:

\[ \omega = (0,0,1)^T \]
Motion Field Equation: Estimate Depth

If $T$, $\omega$, and $f$ are known or measured, then for each image point $(u,v)$, one can solve for the depth $Z$ given measured motion $(du/dt, dv/dt)$ at $(u,v)$.

$$
\begin{align*}
\dot{u} &= \frac{T_y - T_z f}{Z} - \omega_x f + \omega_y \frac{\mu v}{f} - \frac{\omega_z u^2}{f} \\
\dot{v} &= \frac{T_z - T_x f}{Z} + \omega_y f - \omega_x \frac{\mu v}{f} - \frac{\omega_z v^2}{f}
\end{align*}
$$

Pure Rotation: Motion Field on Sphere

Optical Flow