Shape-from-X, Stereo Vision I

Introduction to Computer Vision
CSE 152
Lecture 12

What to do with edges?

• Group unlinked or unrelated edges into line (or curves in general), Hough.

• Segment linked edge chains into curve features (e.g., line segments).

• Accurately fitting parametric curves (e.g., lines) to grouped edge points.

Line Fitting

Problem: minimize

\[ E = \sum_{i=1}^{n} (ax_i + by_i - d)^2 \]

with respect to \((a, b, d)\).

1. Minimize \(E\) with respect to \(d\):

\[ d = \frac{1}{n} \sum_{i=1}^{n} ax_i + by_i \]

2. Substitute \(d\) back into \(E\):

\[ E = \sum_{i=1}^{n} (ax_i + by_i - \frac{1}{n} \sum_{i=1}^{n} ax_i + by_i)^2 \]

where \(n = (a, b)^T\).

3. Minimize \(E = n^TUn = n^TSn\) with respect to \(a, b\) subject to constraint \(n^Tn = 1\).

\[ S = \hat{u}^T \hat{u} = \begin{pmatrix} \sum_{i=1}^{n} x_i^2 - n\bar{x}^2 & \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y} \\ \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y} & \sum_{i=1}^{n} y_i^2 - n\bar{y}^2 \end{pmatrix} \]

where \(S\) is real, symmetric, positive definite.

4. This is a constrained optimization problem in \(n\). Solve with Lagrange multiplier

\[ L(n) = n^TSn - \lambda (n^Tn - 1) \]

Take partial derivative (gradient) w.r.t. \(n\) and set to 0.

\[ \nabla L = 2Sn - 2\lambda n = 0 \]

or

\[ Sn = \lambda n \]

\(n = (a, b)\) is an Eigenvector of \(S\) (the one corresponding to the smallest Eigenvalue).

5. \(d\) is computed from Step 1.

Line Fitting – Finished

Announcements

• Today
  – Line wrapup
  – Shape from X
  – Stereo Vision I
Shape-from-X (i.e., Reconstruction)

- Methods for estimating 3-D shape from image data. X can be one (or more) of many cues.
  - Stereo (two or more views, known viewpoints)
  - Motion (moving camera or object)
  - Shading
  - Changing lighting (Photometric Stereo)
  - Texture variation
  - Focus/blur
  - Prior knowledge/context
  - structured light/lasers

Example: Helmholtz Stereo
Depth + Normals + BRDF

Stereo

Binocular Stereopsis: Mars
Given two images of a scene where relative locations of cameras are known, estimate depth of all common scene points.

Two images of Mars

An Application: Mobile Robot Navigation


The INRIA Mobile Robot, 1990.

Commercial Stereo Heads

Trinocular stereo

Binocular stereo
Stereo can work well

Need for correspondence

Triangulation

Stereo Vision Outline

- Offline: Calibrate cameras & determine “epipolar geometry”
- Online
  1. Acquire stereo images
  2. Rectify images to convenient epipolar geometry
  3. Establish correspondence
  4. Estimate depth

Reconstruction: General 3-D case

• Linear Method: find $P$ such that
  \[ \begin{align*}
  p \times M P &= 0 \\
  P \times M^T P &= 0
  \end{align*} \]
  \[ \iff \begin{align*}
  |p_x| |M| P &= 0 \\
  |p_x| |M^T| P &= 0
  \end{align*} \]

• Non-Linear Method: find $Q$ minimizing
  $\mathcal{E}(p, q) + \mathcal{E}(p', q')$
Two Approaches

- A) From each image, process “monocular” image to obtain cues.
- B) Establish correspondence between cues.
- C) Directly compare image regions between the two images.

Human Stereopsis: Binocular Fusion

How are the correspondences established?

Julesz (1971): Is the mechanism for binocular fusion a monocular process or a binocular one??
- There is anecdotal evidence for the latter (camouflage).

Random Dot Stereograms

A Cooperative Model (Marr and Poggio, 1976)

- Potential matches for \( p \) have to lie on the corresponding epipolar line \( l' \).
- Potential matches for \( p' \) have to lie on the corresponding epipolar line \( l \).
Epipolar Plane  • Baseline
Epipoles
Epipolar Lines