Hough, Lines, Midterm Review

Introduction to Computer Vision
CSE 152
Lecture 10

Announcements

• HW 2 due tomorrow.
• Midterm: Tuesday, May 3.

Physical causes of edges

1. Object boundaries
2. Surface normal discontinuities
3. Reflectance (albedo) discontinuities
4. Lighting discontinuities (shadow boundaries)

Edge is Where Change Occurs: 1-D

• Change is measured by derivative in 1D

- Biggest change, derivative has maximum magnitude
- Or 2nd derivative is zero.

Numerical Derivatives

Take Taylor series expansion of $f(x)$ about $x_0$

$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{1}{2} f''(x_0)(x-x_0)^2 + \cdots$

Consider samples taken at increments of $h$ and first two terms, we have

$f(x_0+h) = f(x_0) + f'(x_0)h + \frac{1}{2} f''(x_0)h^2$

$f(x_0-h) = f(x_0) - f'(x_0)h + \frac{1}{2} f''(x_0)h^2$

Subtracting and adding $f(x_0+h)$ and $f(x_0-h)$ respectively yields

$f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h}$
$f''(x_0) = \frac{f(x_0+h) + f(x_0-h) - 2f(x_0)}{2h^2}$

Convolving with

First Derivative: $[-1 0 1]$
Second Derivative: $[-1 2 -1]$

2D Edge Detection: Canny

1. Filter out noise
   - Use a 2D Gaussian Filter. $J = I \ast G$

2. Take a derivative
   - Compute the magnitude of the gradient:

$$\nabla J = (J_x, J_y) = \left( \frac{\partial J}{\partial x}, \frac{\partial J}{\partial y} \right)$$
is the Gradient

$$\|\nabla J\| = \sqrt{J_x^2 + J_y^2}$$
There are three major issues:
1. The gradient magnitude at different scales is different; which scale should we choose?
2. The gradient magnitude is large along thick trail; how do we identify the significant points?
3. How do we link the relevant points up into curves?

Non-maximum suppression
Using normal at q, find two points p and r on adjacent rows (or columns).
We have a maximum if the value is larger than those at both p and at r.
Interpolate to get values.

Hysteresis Tresholding
- Track edge points by starting at point where gradient magnitude > \( \tau_{\text{high}} \).
- Follow edge in direction orthogonal to gradient.
- Stop when gradient magnitude < \( \tau_{\text{low}} \).
  - i.e., use a high threshold to start edge curves and a low threshold to continue them.

What to do with edges?
- Segment linked edge chains into curve features (e.g., line segments).
- Group unlinked or unrelated edges into lines (or curves in general).
- Accurately fitting parametric curves (e.g., lines) to grouped edge points.

Hough Transform
- [Patented 1962]
Hough Transform: “Noisy line”

- R, θ representation of line
- Maximum accumulator value is 6

Hough Transform: Random points

- R, θ representation of line
- Maximum accumulator value is 4

Mechanics of the Hough transform

- Difficulties
  - How big should the cells be? (too big, and we cannot distinguish between quite different lines; too small, and noise causes lines to be missed)
- How many lines?
  - Count the peaks in the Hough array
- Who belongs to which line?
  - Tag the votes
- Complications, problems with noise and cell size

Number of votes that the real line of 20 points gets with increasing noise

TEM Image of Keyhole Limpet Hemocyanin with detected particles
FIG. Three-dimensional maps of KLH at a resolution of 23.5 Å reconstructed using particles extracted either manually or automatically as described in the text. (a), (b) The side- and top-view of a 3D map reconstructed from a set of 1042 manually selected particle images. (c), (d) The side- and top-view of a 3D map from a set of automatically extracted 1243 particle images.

Processing in Stage 1 for KLH

- Canny edge detection.
- A sequence of ordered Hough transforms (HT’s) is applied in order from the computationally simplest one to the most complex one.
- Edges covered by the detected shapes are removed immediately from edge images following the application of the last HT.

Picking KLH Particles in Stage 1

Zhu et al., IEEE Transactions on Medical Imaging, In press, 2003

Line Fitting

Given n points \((x_i, y_i)\), estimate parameters of line \(ax_i + by_i - d = 0\) subject to the constraint that \(a^2 + b^2 = 1\)

Problem: minimize

\[ E(a, b, d) = \sum_{i=1}^{n} (ax_i + by_i - d)^2 \]

where \((a, b, d)\) is the mean of the data points

\[ \Sigma x_i y_i \]

Sum of squared distances between each point and the line

1. Minimize \(E\) with respect to \(d\): \[ \frac{\partial E}{\partial d} = 0 \Rightarrow d = \frac{1}{n} \sum_{i=1}^{n} ax_i + by_i = \bar{a}x + \bar{b}y \]

Line Fitting – Finished

4. This is a constrained optimization problem in \(n\).

Solve with Lagrange multiplier

\[ L(n) = n^T S n - \lambda (n^T n - 1) \]

Take partial derivative (gradient) w.r.t. \(n\) and set to 0.

\[ \nabla L = 2S n - 2\lambda n = 0 \]

or

\[ S n = \lambda n \]

where \(S\) is real, symmetric, positive definite

5. \(d\) is computed from Step 1.
Midterm
Tuesday, May 3

• In class
• Full period
• Coverage – everything up to this point including readings
• “Cheat sheet” – you can prepare a one sided, 8.5” by 11” sheet of notes. It must be hand written. (After the midterm, save your sheet since you can use the other side for the final).

Incomplete list of topics covered...

• Human visual system
  – Physiology – from eye to brain
  – Phenomenological
  – Function
• Camera models
• Factors in producing images
• Projection models
  – Perspective
  – Orthographic
• Homogenous Coordinates, Vanishing points
• Lenses, Distortion
• Sensors
• Quantization/Resolution
• Illumination
• Reflectance
  – BRDF
  – Lambertian
  – Specular
  – Phong
• Color
  – Light Spectrum
  – Reflectance, source
  – Sensor response
  – Color spaces
  – Chromaticity, YUV, RGB

Topics cont.

• Binary Vision
  – Thresholding
  – Neighborhoods
  – Connected component exploration
  – Features, moments
• Noise
  – Additive, Gaussian noise
• Filtering, linear, convolution with Kernel
  – Averaging/smoothing
  – Sharpening
  – Derivatives
  – Gaussian filter
  – Separability
• Edges & Edge detection
• Edge sources
• Canny
  – Gaussian derivatives
  – Magnitude, orientation
  – Non-maximal supression
  – Linking/thresholding
• Hough Transform
• Generalized Hough transform