Announcements

- Assignment 1: posted to web page, due on Thursday
- Read Trucco & Verri: pp. 15-40

Pinhole Camera: Perspective projection

- Abstract camera model - box with a small hole in it

Geometric Aspects of Perspective Projection

- Points project to points
- Lines project to lines
- Angles & distances (or ratios) are NOT preserved under perspective
- Vanishing point

The equation of projection

Cartesian coordinates:
- We have, by similar triangles, that $(x, y, z) \rightarrow (f x/z, f y/z, -f)$
- Ignore the third coordinate, and get

Euclidean -> Homogenous-> Euclidean

In 2-D
- Euclidean -> Homogenous: $(x, y) \rightarrow k (x,y,1)$
  (can just take $k=1$)
- Homogenous --> Euclidean: $(x, y, z) \rightarrow (x/z, y/z)$

In 3-D
- Euclidean -> Homogenous: $(x, y, z) \rightarrow k (x,y,z,1)$
  (can just take $k=1$)
- Homogenous --> Euclidean: $(x, y, z, w) \rightarrow (x/w, y/w, z/w)$
The camera matrix

Turn

\((x, y, z) \rightarrow \left( \frac{x}{z}, \frac{y}{z} \right)\)

into homogenous coordinates

- HC’s for 3D point are \((X, Y, Z, 1)\)
- HC’s for point in image are \((U, V, W)\)

Affine Camera Model

- Take Perspective projection equation, and perform Taylor Series Expansion about (some point \((x_0, y_0, z_0)\)).
- Drop terms of higher order than linear.
- Resulting expression is called affine camera model.
- Properties
  - Pts. map to pts, lines map to lines
  - Parallel lines map to parallel lines (no vanishing point – at infinity)
  - Ratios of distance/angles preserved

Orthographic projection

Start with affine camera model, and take Taylor series about \((x_0, y_0, z_0) = (0, 0, z_0)\) – a point on optical axis

Depth \((z)\) is lost

The projection matrix for orthographic projection

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Coordinate Changes: Pure Rotations

A rotation matrix \(R\) has the following properties:
- Its inverse is equal to its transpose \(R^T = R^{-1}\)
- Its determinant is equal to 1: \(\det(R) = 1\).
- Or equivalently:
  - Rows (or columns) of \(R\) form a right-handed orthonormal coordinate system.

\[
O_B P = O_B O_A + O_A \Delta P , \quad B P = A P + B O_A
\]
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Rotation: Homogenous Coordinates

- About z axis

$$
\begin{pmatrix}
    x' \\
    y' \\
    z' \\
    1
\end{pmatrix} =
\begin{pmatrix}
    \cos \theta & -\sin \theta & 0 & 0 \\
    \sin \theta & \cos \theta & 0 & 0 \\
    0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
    x \\
    y \\
    z \\
    1
\end{pmatrix}
$$

Roll-Pitch-Yaw

$R = rot(\hat{\theta}, \hat{\beta}, \hat{\phi})$

Euler Angles

$R = rot(\hat{k, \alpha}, \hat{j}, \hat{\beta}, \hat{\theta})$

Rotation

- About $(\hat{k, \hat{k, \hat{k}}})$, a unit vector on an arbitrary axis (Rodrigues Formula)

$$
\begin{pmatrix}
    x' \\
    y' \\
    z'
\end{pmatrix} =
\begin{pmatrix}
    k(k(1-c)+c) & k(k(1-c)-ks) & k(k(1-c)+ks) \\
    k(k(1-c)+ks) & k(k(1-c)+c) & k(k(1-c)-ks) \\
    k(k(1-c)-ks) & k(k(1-c)-ks) & k(k(1-c)+c)
\end{pmatrix}
\begin{pmatrix}
    x \\
    y \\
    z
\end{pmatrix}
$$

where $c = \cos \theta$ & $s = \sin \theta$

Coordinate Changes: Rigid Transformations

$$
B P = B R A P + B O_A
$$
Block Matrix Multiplication

\[ A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, \quad B = \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix} \]

What is \( AB \)?

\[ AB = \begin{bmatrix} A_1B_1 + A_2B_2 \\ A_3B_1 + A_4B_2 \end{bmatrix} = \begin{bmatrix} A_1 \cdot B_1 + A_2 \cdot B_2 \\ A_3 \cdot B_1 + A_4 \cdot B_2 \end{bmatrix} \]

Homogeneous Representation of Rigid Transformations

\[ \begin{bmatrix} p' \\ 1 \end{bmatrix} = \begin{bmatrix} R & t' \\ 0 & 1 \end{bmatrix} \begin{bmatrix} p \\ 1 \end{bmatrix} \]

Transformation represented by 4 by 4 Matrix

Camera parameters

- Issue
  - camera may not be at the origin, looking down the z-axis
    - extrinsic parameters (Rigid Transformation)
  - one unit in camera coordinates may not be the same as one unit in world coordinates
    - intrinsic parameters - focal length, principal point, aspect ratio, angle between axes, etc.

\[ \begin{bmatrix} U' \\ V' \\ W' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} \]

Camera Calibration

Given \( n \) points \( P_1, \ldots, P_n \) with known positions and their images \( p_1, \ldots, p_n \), estimate intrinsic and extrinsic camera parameters

- See Text book for how to do it.

Getting more light – Bigger Aperture

Pinhole Camera Images with Variable Aperture

2 mm

1 mm

.6 mm

.35 mm

.15 mm

.07 mm

Limits for pinhole cameras

2.38. EXHIBITION LIMITS THE QUALITY OF PINHOLE OPTICS. These three images of a Luke format were made using pinholes with increasing size. (a) When the pinhole is relatively large, the image is not properly convoluted, and the image is blurred. (b) Reducing the size of the pinhole improves the focus. (c) Reducing the size of the pinhole further sharpens the focus, due to diffraction, from Pinchord, 1918.
The reason for lenses

Thin Lens

- Rotationally symmetric about optical axis.
- Spherical interfaces.

Thin Lens: Center

• All rays that enter lens along line pointing at \( O \) emerge in same direction.

Thin Lens: Focus

Parallel lines pass through the focus, \( F \)

Thin Lens: Image of Point

All rays passing through lens and starting at \( P \) converge upon \( P' \)

Thin Lens: Image of Point

\[
\frac{1}{z'} - \frac{1}{z} = \frac{1}{f}
\]
**Thin Lens: Image Plane**

A price: Whereas the image of $P$ is in focus, the image of $Q$ isn’t.

**Thin Lens: Aperture**

- Smaller Aperture -> Less Blur
- Pinhole -> No Blur

**Field of View**

Deviations from the lens model

Deviations from this ideal are **aberrations**

Two types

1. geometrical
   - spherical aberration
   - astigmatism
   - distortion
   - coma

2. chromatic

Aberrations are reduced by combining lenses

**Compound lenses**

**Spherical aberration**

rays parallel to the axis do not converge
outer portions of the lens yield smaller focal lengths

**Distortion**

magnification/focal length different for different angles of inclination

Can be corrected! (if parameters are known)
Chromatic aberration

Index of refraction of lens depends on wavelength of light

Chromatic aberration

Rays of different wavelengths focus in different planes

Cannot be removed completely

Sometimes achromatization is achieved for more than 2 wavelengths

Vignetting in Compound Lenses

Radiometry, Lighting, Intensity

Lighting

- Applied lighting can be represented as a function on the 4-D ray space (radiances)
- Special light sources
  - Point sources
  - Distant point sources
  - Strip sources
  - Area sources
- Common to think of lighting at infinity (a function on the sphere, a 2-D space)

Irradiance

- How much light is arriving at a surface?
- Irradiance -- power per unit area W/cm²
- Total power arriving at the surface is given by adding irradiance over all incoming angles
Camera’s sensor

- Measured pixel intensity is a function of irradiance integrated over
  - pixel’s area
  - over a range of wavelengths
  - For some time

\[
I = \iiint E(x, y, \lambda, t)x(x, y)q(\lambda)dxdydtdt
\]

Light at surfaces

Many effects when light strikes a surface -- could be:

- transmitted
  - Skin, glass
- reflected
  - mirror
- scattered
  - milk
- travel along the surface and leave at some other point
- absorbed
  - sweaty skin

Assume that

- surfaces don’t fluoresce
  - e.g. scorpions, detergents
- surfaces don’t emit light (i.e. are cool)
- all the light leaving a point is due to that arriving at that point

BRDF

- Bi-directional Reflectance Distribution Function
  \[\rho(\theta_{in}, \phi_{in}, \theta_{out}, \phi_{out})\]
- Function of
  - Incoming light direction: \(\theta_{in}, \phi_{in}\)
  - Outgoing light direction: \(\theta_{out}, \phi_{out}\)
- Ratio of incident irradiance to emitted radiance

Surface Reflectance Models

Common Models

- Lambertian
- Phong
- Physics-based
  - Specular [Blinn 1977], [Cook-Torrance 1982], [Ward 1992]
  - Diffuse [Hanrahan, Kreuger 1993]
  - Generalized Lambertian [Oren, Nayar 1995]
  - Thoroughly Pitted Surfaces [Koenderink et al 1999]
- Phenomenological [Koenderink, Van Doorn 1996]

Arbitrary Reflectance

- Non-parametric model
- Anisotropic
- Non-uniform over surface
- BRDF Measurement [Dana et al, 1999], [Marschner]

Lambertian Surface

At image location \((u,v)\), the intensity of a pixel \(x(u,v)\) is:

\[
x(u,v) = [a(u,v) \cdot n(u,v)] \cdot s_0
\]

where

- \(a(u,v)\) is the albedo of the surface projecting to \((u,v)\).
- \(n(u,v)\) is the direction of the surface normal.
- \(s_0\) is the light source intensity.
- \(s\) is the direction to the light source.

Specular Reflection: Smooth Surface

Phong – rough, specular
Color Cameras

We consider 3 concepts:

1. Prism (with 3 sensors)
2. Filter mosaic
3. Filter wheel

…and X3

Prism color camera

- Separate light in 3 beams using dichroic prism
- Requires 3 sensors & precise alignment
- Good color separation

Coat filter directly on sensor

Demosnaicing (obtain full colour & full resolution image)

new color CMOS sensor
Foveon’s X3

better image quality
smarter pixels
Robust Color Blob Tracking

Color Blob tracking

• Color-based tracker gets lost on white knight: Same Color

The appearance of colors

• Color appearance is strongly affected by (at least):
  – Spectrum of lighting striking the retina
  – other nearby colors (space)
  – adaptation to previous views (time)
  – "state of mind"

From Foundations of Vision, Brian Wandell, 1995, via B. Freeman slides