Continuous Motion

Introduction to Computer Vision
CSE 152
Lecture 16

Some counting
Consider $M$ images of $N$ points, how many unknowns?
1. Affix coordinate system to location of first camera location: $(M-1)*6$ Unknowns
2. 3-D Structure: $3N$ Unknowns
3. Can only recover structure and motion up to scale. Why?

Total number of unknowns: $(M-1)*6+3N-1$

The Eight-Point Algorithm (Longuet-Higgins, 1981)

Set $F_{33}$ to 1

Solve

For $F$

Detection of Corner Features

• Need two strong edges:
• Example:

Create the following matrix:

If $\min(\lambda_1, \lambda_2) > T$

There is a corner!

Eigenvalues of $C$

(Or create an matrix with the same dimension as an image with value of $\min(\lambda_1, \lambda_2)$ at each location–Find local maximum)
Feature matching

Evaluate normalized cross correlation (or sum of squared differences) for all features with similar coordinates

e.g. \((x', y') = [x' = x + \Delta x, y' = y + \Delta y]\)

Keep mutual best matches

Still many wrong matches!

Reconstruction Results (Tomasi and Kanade, 1992)

Fiat Lux:
An Application of Discrete SFM

Building Blocks

- For more info, see http://www.cs.berkeley.edu/~Debevec/
  http://www.cs.berkeley.edu/~Debevec/Items/NewScientist/

- Structure and Motion from Line Segments in Multiple Images, C. J. Taylor, D. Kriegman, IEEE PAMI, 1995
- Facade: Modeling and Rendering Architecture from Photographs, P. Debevec, C.J. Taylor, J. Malik, SIGGRAPH 96
- Recovering High Dynamic Range Radiance Maps from Photographs, P. Debevec, J. Malik SIGGRAPH 97
- Rendering Synthetic Objects into Real Scenes, P. Debevec, SIGGRAPH 98

St. Peter’s Plan

Model of St. Peter’s constructed with Facade
Comparison of measured segment to projected line

Images with marked features

Resulting model & Camera Positions

Recovered model edges reprojected through recovered camera positions into the three original images
View-Dependent texture mapping

Composite of all texture maps

High Dynamic Range Radiance Maps

St. Peter’s Panorama

Creating the Radiance Map

Two images of a two-inch mirrored sphere placed in front of Bernini’s Baldacchino inside St. Peter’s

FIAT LUX Radiance Maps

St. Peters 200,000:1
Panoramic environment created by taking multiple radiance images and assembling them into panoramas. Used to create the background plates for the film. Three-dimensionality was added to these backgrounds by projecting them onto a model of the corresponding environments.

Continuous Motion

- Consider a video camera moving continuously along a trajectory (rotating & translating).
- How do points in the image move?
- What does that tell us about the 3-D motion & scene structure?

Is motion estimation inherent in humans?
Demo

Motion

“When objects move at equal speed, those more remote seem to move more slowly.”
- Euclid, 300 BC

Motion Reveals

1. Object boundaries – segmentation
2. Abrupt changes to scenes – movie shot?
3. Observer motion
4. 3-D structure
Simplest Idea for video processing

Image Differences
- Given image $I(u,v,t)$ and $I(u,v, t+\delta t)$, compute $I(u,v, t+\delta t) - I(u,v,t)$.
- This is partial derivative: $\frac{\partial I}{\partial t}$
- At object boundaries, $\frac{\partial I}{\partial t}$ is large and is cue for segmentation
- Doesn’t tell which way stuff is moving

Background Subtraction
- Gather image $I(x,y,t_0)$ of background without objects of interest (perhaps computed over average over many images).
- At time $t$, pixels where $|I(x,y,t)-I(x,y,t_0)| > \tau$ are labeled as coming from foreground objects.

The Motion Field

Where in the image did a point move?

Down and left

THE MOTION FIELD

The “instantaneous” velocity of points in an image

LOOMING

The Focus of Expansion (FOE)

Intersection of velocity vector with image plane

With just this information it is possible to calculate:
1. Direction of motion
2. Time to collision

Rigid Motion: General Case

Position and orientation of a rigid body

Rotation Matrix & Translation vector

Angular Velocity Vector: $\omega$ (or $\Omega$)

\[
\dot{p} = T + \omega \times p
\]
General Motion

\[
\begin{bmatrix}
  u \\
  v \\
  \dot{u} \\
  \dot{v}
\end{bmatrix} =
\begin{bmatrix}
  f & 0 & 0 & 0 \\
  0 & f & 0 & 0 \\
  \frac{f}{z} & 0 & 1 & 0 \\
  0 & \frac{f}{z} & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  z \\
  \dot{x} \\
  \dot{y} \\
  \dot{z}
\end{bmatrix}
\]

Substitute \( \dot{p} = T + \omega \times p \) where \( p = (x,y,z)^T \)

Motion Field Equation

\[
\begin{aligned}
\dot{u} &= \frac{T_{u}v - T_{v}f}{Z} - \omega_{x}f + \omega_{y}v + \frac{\omega_{u}v}{f} - \frac{\omega_{v}u^{2}}{f} \\
\dot{v} &= \frac{T_{v}u - T_{u}f}{Z} + \omega_{x}f - \omega_{y}u - \frac{\omega_{u}u}{f} - \frac{\omega_{v}v^{2}}{f}
\end{aligned}
\]

- \( T \): Components of 3-D linear motion
- \( \omega \): Angular velocity vector
- \( (u,v) \): Image point coordinates
- \( Z \): depth
- \( f \): focal length

Pure Translation

\[
\begin{aligned}
\dot{u} &= \frac{T_{u}v - T_{v}f}{Z} - \omega_{x}f + \omega_{y}v + \frac{\omega_{u}v}{f} - \frac{\omega_{v}u^{2}}{f} \\
\dot{v} &= \frac{T_{v}u - T_{u}f}{Z} + \omega_{x}f - \omega_{y}u - \frac{\omega_{u}u}{f} - \frac{\omega_{v}v^{2}}{f}
\end{aligned}
\]

\[ \omega = 0 \]

Forward Translation & Focus of Expansion

[Gibson, 1950]

Parallel (FOE point at infinity)

Parallel

Sideways Translation

[Gibson, 1950]

Parallel (FOE point at infinity)

Parallel
Pure Rotation: $T=0$

$$\begin{align*}
\dot{u} &= -u - T_x f - \omega_x f + \omega_y v + \frac{\omega_x v u}{f} - \frac{\omega_y u^2}{f} \\
\dot{v} &= -v - T_y f + \omega_y f - \omega_x u - \frac{\omega_x u v}{f} - \frac{\omega_y v^2}{f} \\
\end{align*}$$

- Independent of $T_x$, $T_y$, $T_z$
- Independent of $Z$
- Only function of $(u,v)$, $f$, and $\omega$

Motion Field Equation: Estimate Depth

$$\begin{align*}
\dot{u} &= \frac{T_{z} - u T_{f}}{Z} - \omega_z f + \omega_y v + \frac{\omega_x v u}{f} - \frac{\omega_y u^2}{f} \\
\dot{v} &= \frac{T_{y} - v T_{f}}{Z} + \omega_y f - \omega_x u - \frac{\omega_x u v}{f} - \frac{\omega_y v^2}{f} \\
\end{align*}$$

If $T$, $\omega$, and $f$ are known or measured, then for each image point $(u,v)$, one can solve for the depth $Z$ given measured motion $(\frac{du}{dt}, \frac{dv}{dt})$ at $(u,v)$.

Rotational MOTION FIELD

The "instantaneous" velocity of points in an image.

PURE ROTATION

$\omega = (0,0,1)^T$