Question 1

Consider a plane with surface normal \( \mathbf{n} \) and distance \( d \) from the origin and given by equation \( \mathbf{n} \cdot \mathbf{p} = d \) where \( \mathbf{n} = (nx, ny, nz) \) and \( \mathbf{p} = (x, y, z) \). Let the camera have an optical axis in the z direction and focal length \( f \). Let the camera be moving with translation \( \mathbf{T} = (Tx, Ty, Tz) \) and angular velocity \( (\omega_x, \omega_y, \omega_z) \). Show that the optical flow field is the following quadratic function of \((x, y)\)

\[
\begin{align*}
v_x &= \frac{1}{f^2} \left( a_1 x^2 + a_2 xy + a_3 fx + a_4 fy + a_5 f^2 \right) \\
v_y &= \frac{1}{f^2} \left( a_1 xy + a_2 y^2 + a_6 fy + a_7 fx + a_8 f^2 \right)
\end{align*}
\]

where

\[
\begin{align*}
a_1 &= -d \omega_y + T_z n_x \\
a_2 &= d \omega_x + T_z n_y \\
a_3 &= T_z n_z - T_x n_x \\
a_4 &= d \omega_z - T_x n_y \\
a_5 &= -d \omega_y - T_x n_z \\
a_6 &= T_z n_z - T_y n_y \\
a_7 &= -d \omega_z - T_y n_x \\
a_8 &= d \omega_x - T_y n_z
\end{align*}
\]
Question 2 Photometric Stereo: Consider a Lambertian surface of constant albedo and a unit strength light source so that the image formation equation is \( E = \mathbf{n} \cdot \mathbf{s} \). Suppose that two measurements, \( E_1 \) and \( E_2 \) are taken with two light source positions, \( \mathbf{s}_1 \) and \( \mathbf{s}_2 \), determine the two possible solutions for the unit surface normal. When is there only one solution?

Question 3 Optical flow: Consider a camera modelled by orthographic projection and a piece of paper parallel to the image plane with an albedo pattern illuminated so that: \( E(x, y) = \sin(2y)\cos(x) \). Let the camera translate parallel to the image plane with velocity vector \( \mathbf{v} = (v_x, v_y) \).

1. Using the brightness constancy assumption, what is the normal component of the optical flow? (i.e., \( \mathbf{v} \cdot \mathbf{n} \) in equation (8.18) in the text?)
2. Is there a point \( (x, y) \) where the normal component is zero?
3. For what values of \( (x, y) \) does it equal \( \mathbf{v} \)?