CSE 101 Homework 2
Due Tuesday, May 11
Divide-and-Conquer
100 points total = 10 %

**Binary Conversion: 20 pts.** In the calibration homework, we found an $O(n^2)$ algorithm for converting a decimal integer to binary, where the basic operations involved single digits. Present and analyze a divide-and-conquer algorithm that does better. Use the faster integer multiplication algorithm, *Mult*, shown in class, which runs in time $O(n^{\log_2 3})$ as a subroutine.

**Median of two sorted lists: 20 pts.** The median of a set of numbers is an element of that set so that half the elements (round down) are less than that number and half are at least as large as that number. Present and analyze a divide-and-conquer algorithm that, given two sorted arrays of distinct integers, $A[1..n], B[1..n]$, returns the median ($n$'th largest) of the set of elements that appear in either list.

**Multiplication in threes-20pts.** Describe and analyze a divide-and-conquer algorithm for integer multiplication based on partitioning up each input into 3 blocks of digits. Is this better than the divide and conquer algorithm from class?

**Skylines:(20 points)** Present and analyze a divide-and-conquer algorithm for the Skylines problem discussed in class. As a reminder, the problem was as follows. In a video game, a city’s buildings are rectangles along an axis, specified as $B_i = (s_i, f_i, h_i)$, for $1 \leq i \leq n$, for a building starting at $x = s_i$ and going to $x = f_i$ at height $h_i$. When seen from a distance, the graphics is supposed to plot the skyline, which at each vertical point $x$ has height the height of the tallest building $B_i$ with $s_i \leq x \leq f_i$. Give an algorithm to plot the skyline, as a series of points connected by line segments.

**Implementation of Skylines:(20 points)** Implement the above divide and conquer algorithm for the Skylines problem, and the algorithm using a heap shown in class. Compare the running times (graph of a log-log scale) for different values of $n$. For each value, generate $n$ random buildings $(s, f, h)$ by picking $s$ randomly between 1 and $n^2$, pick $f = s + 2^i$, where $i$ is chosen randomly between 0 and $\log n$, and pick $h$ randomly between 1 and $n$. Plot the average time for several random instances with $n$ all powers of 2 between $2^4$ and $2^{12}$.