On this exam you are allowed to use a calculator and two 8.5" by 11" sheets of notes. The total number of points possible is 45. In order to get full credit you must show all your work. Good luck!

1. (3 pts.) Consider the two lines \( x = 1 \) and \( x = 2 \) in \( \mathbb{P}^2 \). Write down the expression for each line (1 and 1') in homogeneous notation and solve for their point of intersection.

2. (12 pts. = 4 \times 3 pts.) Let the following matrix \( H \) denote a general 2D planar transformation:

\[
H = \begin{bmatrix}
    h_{11} & h_{12} & h_{13} \\
    h_{21} & h_{22} & h_{23} \\
    h_{31} & h_{32} & h_{33}
\end{bmatrix}
\]

For each of the following special cases, write down (i) the parameterization of the \( h_{ij} \)'s, (ii) the number of degrees of freedom, and (iii) the number of correspondences needed to estimate \( H \).

(a) Euclidean
(b) Similarity
(c) Affine
(d) Projective

3. (3 pts.) Write down the essential matrix \( (E) \) and the epipoles \( (e \text{ and } e') \) for the special case of a camera undergoing pure translation with \( t = (1, 0, 0)^T \).

4. (3 pts.) What can be seen in three dimensions from an uncalibrated stereo rig?

5. (3 pts.) Which eigenvector of the normalized affinity matrix \( P = D^{-1}W \) does Normalized Cut use to find an approximately optimal bipartition? What is the significance of the corresponding eigenvalue?

6. (3 pts.) Helmholtz reciprocity provides a constraint on the BRDF. Explain this principle in words and write down the condition that \( \rho_{bd}(\theta_o, \phi_o, \theta_i, \phi_i) \) must satisfy as a consequence.

7. (2 pts.) Why are the filters used in texture analysis often based on 2nd and higher directional derivatives of Gaussians?

8. (2 pts.) What assumption do Tomasi and Kanade make about the camera in their “Factorization Method” for estimating structure from motion? As a rule of thumb, when is this assumption valid?
9. (14 pts.) In this problem, your task will be to devise a system for solving a real-world problem. The problem is as follows. You are given a pair of images containing multiple objects that have undergone motion from one frame to the next. (See Figure 1 for two examples.) Your objective is to segment the image sequence into a finite number of “motion layers.” A motion layer is a subset of pixels in the pair of frames that move together according to a given motion model.

Use the following guidelines in formulating your solution:

- Identify the subproblems.
- Solve each subproblem.
- State your assumptions.
- Address the problems of occlusion, outliers, and noise.
- Explain the limitations of your system.
- Cite relevant references.

Note that once you have identified the subproblems, you can solve them in any order you wish, assuming a reasonable input from the previous stage. You do not need to develop your answer to the point of providing pseudocode, but you should clearly explain which algorithms you are using and what purpose they serve, as well as what the inputs and outputs are for each stage.

Figure 1: Examples of pairs of image frames containing moving objects.