Optical Flow

Computer Vision
CSE 190-B
Lecture 17

Announcements

– Homework 3 due today
  • Typo: Part V, first line - ‘Write up a page or two
description of how you implemented the algorithm in Part
II...’ It should be part III.
– Thursday, May 29 - No class
– Makeup class, Thursday, June 5, 11:00-12:30,
  APM 4218
– Fourth (last) homework assigned today
– Final Exam, Thursday, June 12, 3:00-6:00,  WLH
  2205

Outline

• Today
  – Infinitesimal motion field
  – Optical flow

Motion

Some problems of motion
1. Correspondence: Where have elements of the image
   moved between image frames
2. Reconstruction: Given correspondence, what is 3-D
   geometry of scene
3. Segmentation: What are regions of image corresponding
to different moving objects
4. Tracking: Where have objects moved in the image?
   related to correspondence and segmentation.

Variations:
  – Small motion (video),
  – Wide-baseline (multi-view)

Motion Demo
Motion

“When objects move at equal speed, those more remote seem to move more slowly.”
- Euclid, 300 BC

The Motion Field

Where in the image did a point move?

Down and left

The Motion Field

The Motion Field Equation

\[ v_x = \frac{T_x - T_x f}{Z} - \omega_y f + \omega_x y + \frac{\omega_y x y}{f} \]
\[ v_y = \frac{T_y - T_y f}{Z} + \omega_x f - \omega_x y - \frac{\omega_x y^2}{f} \]

- T: Components of 3-D linear motion
- \( \omega \): Angular velocity vector
- (x,y): Image point coordinates
- Z: depth
- f: focal length

Pure Translation

Forward Translation & Focus of Expansion

[Gibson, 1950]
Pure Translation

Radial (FOE point at infinity)

\( T_x = 0 \)

Motion parallel to image plane

Parallel

Sideways Translation

[Gibson, 1950]

Pure Rotation: \( T=0 \)

\[
\begin{align*}
v_x &= \frac{T_x - T_x f}{Z} - \omega_y f + \omega_z y + \frac{\omega_x x y}{f} - \frac{\omega_y x^2}{f} \\
v_y &= \frac{T_y - T_y f}{Z} + \omega_z f - \omega_z x - \frac{\omega_x x y}{f} + \frac{\omega_y y^2}{f}.
\end{align*}
\]

- Independent of \( T_x, T_y, T_z \)
- Independent of \( Z \)
- Only func of \((x,y), \tau, \text{ and } \omega\)

Motion Field Equation: Estimate Depth

\[
\begin{align*}
v_x &= \frac{T_x - T_x f}{Z} - \omega_y f + \omega_z y + \frac{\omega_x x y}{f} - \frac{\omega_y x^2}{f} \\
v_y &= \frac{T_y - T_y f}{Z} + \omega_z f - \omega_z x - \frac{\omega_x x y}{f} + \frac{\omega_y y^2}{f}.
\end{align*}
\]

If \( T, \omega, x, y \) and \( f \) are known (measured), then one can solve for the depth, \( Z \).

Estimating the motion

1. Feature-based (Sect. 8.4.2)
   1. Detect Features (corners) in an image
   2. Search for the same features nearby (Feature tracking).
2. Differential techniques (Sect. 8.4.1)
What is the correspondence of P & P’

Contour plots of image intensity in two images

Motion Field Equation

\[
\begin{align*}
    v_x &= \frac{T_x x - T_x f}{Z} - \omega_y f + \omega_x y + \frac{\omega_x y}{f} - \frac{\omega_y x}{f} \\
    v_y &= \frac{T_y y - T_y f}{Z} + \omega_x f - \omega_x x - \frac{\omega_x y}{f} + \frac{\omega_y x}{f}.
\end{align*}
\]

- \( T \): Components of 3-D linear motion
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Normal Flow

Illusion Works Barber Pole Illusion

Batch Processing of sequences: Factorization

Four of 150 input images

Tracked Corner Features
3-D Reconstruction

Shape Reconstruction: from multiple images

Work by Paul Debevec, C.J. Taylor and Jitendra Malik

Fiat Lux