Notes for week 7 part 2 (Multimedia Servers)

Maximum Users in a Video Server System

Let:
\[
\alpha = 13\text{ms} \\
\beta = 4\text{ms} \\
\gamma = 33\text{ms} \\
\omega = 3\text{ms} \\
\ell_{ds} = 1\text{ms}
\]

First, we’ll compute the maximum number of users that a server with those properties could support. Using the equation:

\[
k \geq \frac{n \times (\alpha - \beta)}{\gamma - n \times \beta}
\]

we can compute the number of blocks each user will need to read in a round. To find the maximum number of users allowable, the denominator of the equation needs to be solved for 0 (this is where we’d need to read an infinite number of blocks to allow that many users in the system):

\[
\gamma - n \times \beta = 0
\]

\[
n = \frac{\gamma}{\beta} = \frac{33}{4} = 8.25
\]

Taking the floor of the computer value for n gives us the maximum users allowed. In this case, it’s 8. We can now plug this value into the equation for k to determine how many blocks a user must read each round:

\[
k \geq \frac{8 \times (13 - 4)}{33 - 8 \times 4} = \frac{72}{1} = 72
\]

Transient Continuity

When new users enter the system, delays will occur in the availability of blocks to each user for the first round after the user has entered the system. Using the system properties above, let’s consider the case where the number of users increases from 5 to 6 (the number of blocks needed per user, k, will increase from 4 to 6).

When \(n=5\), the total time to transfer the data blocks is:
\[
5 \times 13 + 5 \times 3 \times 4 = 125\text{ms} \quad (\text{from } n \times \alpha + n \times (k-1) \times \beta \text{ in the previous notes})
\]

and the total display time per user is:
\[
4 \times 33 = 132\text{ms} \quad (\text{from } k \times \gamma \text{ in the previous notes})
\]
So, with 5 users there are no problems in the system (the total transfer time is less than or equal to the display time per user). When the sixth user is added, during the first round each user will still only have 132ms of data. But, the total transfer time has increased to: $6*13 + 6*5*4 = 198$ms

Now there’s a problem. The transfer time exceeds the display time, so the users will see a discontinuity in their video display whenever a new user joins the system.

The way to combat this is to have an extra block reserved each round. So in our equation:

$n*\alpha + n*(k-1)\beta \leq k*\gamma$

switch $(k-1)$ to $k$ to reflect the extra block:

$n*\alpha + n*k\beta \leq k*\gamma$

So now, when a new user enters the system, one extra block of data is immediately available to be read without causing a pause in the video. Since our example had $n$ increase from 5 to 6, which resulted in $k$ increasing from 4 to 6, we would need to take two rounds before the new user could be added.

Round 1 ($6^{th}$ user has requested data): Users 1-5 read in $k=k+1$ blocks (5)
Round 2 ($6^{th}$ user is waiting): Users 1-5 read in $k=k+1$ blocks (6)
Round 3 ($6^{th}$ user joins): Users 1-6 read in $k=6$ blocks