Given: $\Sigma = \{0, 1\}$.

Two languages we had in 4/2's session are:
$L_1 = \{ \omega \mid \omega \text{ contains the substring 101} \}$, and $L_4 = \{ \omega \mid \omega \text{ contains exactly two } 1's \}$.

Let $M_1$ recognize $L_1$, where $M_1 = (Q_1, \Sigma, \delta_1, q_0, F_1)$, and
$M_4$ recognize $L_4$, where $M_4 = (Q_4, \Sigma, \delta_4, p_0, F_4)$.

$F_1 = \{q_3\}$ and $F_4 = \{p_2\}$.

We construct a DFA, called $M$, to recognize $L_1 \cup L_4$, where $M = (Q, \Sigma, \delta, r_0, F)$.
From page 46 of the text book (Th 1.12 proof):
1. $Q = \{(r_1, r_2) \mid r_1 \in Q_1 \text{ and } r_2 \in Q_4\}$, we name the states $r_0 \ldots r_{15}$.
2. $\Sigma$ is the same.
3. $\delta((r_1, r_2), a) = (\delta_1(r_1, a), \delta_4(r_2, a))$
4. $r_0 = (q_0, p_0)$
5. $F = \{(r_1, r_2) \mid r_1 \in F_1 \text{ or } r_2 \in F_4\} = \{r_2, r_6, r_{10}, r_{12}, r_{13}, r_{14}, r_{15}\}$