Problem 1

When you are looking for the product-of-sums, first you use the K-map to find the sum-of-products for $F'$ and then you complement the result. In this case for $F'$ we have:

Primes: (4,5,6,7), (4,6,12,14), (0,4,8,12), (0,1,4,5)

Essential Primes: (4,5,6,7), (4,6,12,14)

There are 2 different ways to write down $F'$:

i) $F' = (4,5,6,7) + (4,6,12,14) + (0,4,8,12) = a'b + bd' + c'd'$

ii) $F' = (4,5,6,7) + (4,6,12,14) + (0,1,4,5) = a'b + bd' + a'c'$

So finally we get $F$ in the product-of-sums form by complementing $F'$:

i) $F = (a + b')(b' + d)(c + d)$

ii) $F = (a + b')(b' + d)(a + c)$

Problem 2

The first step of the Quine-McCluskey method is to find all the prime implicants of the function:

<table>
<thead>
<tr>
<th></th>
<th>0000</th>
<th>(0,1): 000-</th>
<th>0001</th>
<th>(0,2): 00-0</th>
<th>0010</th>
<th>(0,4): 0-00</th>
<th>0100</th>
<th>(1,5): 0-01</th>
<th>0101</th>
<th>(1,9): -001</th>
<th>(0,1,4,5): 0-0-</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0001</td>
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<td>0010</td>
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<td>0100</td>
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<td>0101</td>
<td>-&gt; (1,9): -001</td>
<td>-&gt; (0,1,4,5): 0-0-</td>
<td>0111</td>
<td>(5,7): 01-1</td>
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<td>1001</td>
<td>(7,15): -111</td>
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<td>1011</td>
<td>(9,11): 10-1</td>
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<td>15</td>
<td>1111</td>
<td>(11,15): 1-11</td>
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</table>

From the first chart we eliminate (5,7) because it is a proper subset of (7,15) and it covers the same number of literals. Then the primary essential primes are:
After removing the necessary lines and columns, the implication chart looks like this:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>7</th>
<th>9</th>
<th>11</th>
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<tbody>
<tr>
<td>(0,2)</td>
<td>v</td>
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<td>(1,9)</td>
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<td>(11,15)</td>
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<tr>
<td>(0,1,4,5)</td>
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<td>v</td>
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</tbody>
</table>

(0,2) and (7,15)

After removing the necessary lines and columns, the implication chart looks like this:

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<tr>
<th></th>
<th>1</th>
<th>9</th>
<th>11</th>
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</thead>
<tbody>
<tr>
<td>(1,9)</td>
<td>v</td>
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<td>(9,11)</td>
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<tr>
<td>(0,1,4,5)</td>
<td>v</td>
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</table>

Now the first thing to do is to eliminate (11,15) as a proper subset of (9,11) covering the same number of literals. Note that (0,1,4,5) is also a proper subset of (1,9), but we don’t eliminate yet, because it covers more literals. Now the secondary essential prime is:

(9,11)

Finally we end up with the selection between (0,1,4,5) and (1,9) to cover minterm 1, in which case we select the first one, because it requires less literals.

So the function can be written like this:

\[ F = (0,2) + (7,15) + (9,11) + (0,1,4,5) = a'b'd' + bcd + ab'd + a'c' \]
**Problem 3**

To solve this problem in an easy way, just break it into intermediate XOR sums:

\[ S_1 = a \text{ XOR } (a'b) \]
\[ S_2 = S_1 \text{ XOR } (a' + b') \]
\[ F = S_2 \text{ XOR } (a + b + c') \]

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<thead>
<tr>
<th></th>
<th>( a )</th>
<th>( b )</th>
<th>( c )</th>
<th>( a )</th>
<th>( a'b )</th>
<th>( S_1 )</th>
<th>( a'+b' )</th>
<th>( S_2 )</th>
<th>( a+b+c' )</th>
<th>( F )</th>
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So \( F \) can be written as:

i) \( F = a'b + ab' + a'c \)
ii) \( F = a'b + ab' + b'c \)

**Problem 4**

\[ Q_0* = T_0 \text{ XOR } Q_0 = Q_0 \text{ XOR } (x'Q_1) = x'Q_0'Q_1 + xQ_0 + Q_0Q_1' \]
\[ Q_1* = T_1 \text{ XOR } Q_1 = Q_1 \text{ XOR } (x + Q_0') = x'Q_0Q_1 + xQ_1' + Q_0'Q_1' \]
### Problem 5

#### Truth Table

<table>
<thead>
<tr>
<th>Q0</th>
<th>Q1</th>
<th>x=0</th>
<th>x=1</th>
<th>S</th>
<th>x=0</th>
<th>x=1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>01</td>
<td>01</td>
<td>0</td>
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<td>1</td>
<td>0</td>
<td>C</td>
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<td>01</td>
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<td>1</td>
<td>0</td>
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</tr>
</tbody>
</table>

#### State Transition Diagram

![State Transition Diagram](image)

#### Circuit Diagram

![Circuit Diagram](image)