Quiz 1 Solutions

CSE 101, Spring 202

Problem 1

The argument fails for the inductive step going from $n = 1$ to $n = 2$. This is because the groups of 1 through $n - 1$ and 2 through n don't have any horses in common.

Problem 2

This problem can best be solved by pairing as Gauss would do (according to legend.) Observe that $101 + 200 = 102 + 199 = \ldots = 301$. There are 50 such of these pairs, so we can calulate the sum of $101 + \ldots 200$, with $50 \times 301 = 15,050$. Now we just need to add the first 100 of the sum to get the solution: 15,150.

Problem 3

Observe that the sum can be grouped in the following manner:

$$
\begin{aligned}
& 1 - 2 + 3 - 4 + 5 + \ldots + 95 - 96 + 97 - 98 + 99 \\
= \; & 1 + 3 + 5 + 7 + \ldots + 93 + 95 + 97 + 99 \\
& -2 - 4 - 6 - 8 - \ldots - 94 - 96 - 98 \\
= \; & \Sigma_{i=1}^{50}(2i - 1) - \Sigma_{1}^{49} 2i \\
= \; & 2\Big(\frac{(50)(51)}{2}\Big) - 50 - 2\Big(\frac{(49)(50)}{2}\Big) \\
= \; & 50
\end{aligned}
$$

Problem 4

Perhaps the best way to solve this recurrence is to examine the first couple of cases and use this to formulate a guess. Once we have the guess we can prove it correct by induction. Note that this is different from the technique used by Prof. Pevzner in class.

$$
\begin{aligned}
T_1 &= 1 \\
T_2 &= 5 \\
T_3 &= 17 \\
T_4 &= 53
\end{aligned}
$$

This gives us $T_n = 2 \cdot 3^{n-1} - 1$.
Base Case: $T_1 = 2 \cdot 3^0 - 1 = 1$
Induction Step:

$$\begin{aligned}
\text{Assume } T_n &= 2 \cdot 3^{n-1} - 1 \\
T_{n+1} &= 3T_n + 2 \\
&= 3(2 \cdot 3^{n-1} - 1) + 2 \\
&= 2 \cdot 3 \cdot 3^{n-1} - 3 + 2 \\
&= 2 \cdot 3^n - 1
\end{aligned}$$