Midterm Practice Problems

CSE 101, Spring 2002

1 Recurrence Relations

Give tight bounds for the following recurrence relations:

- $T(n) = T(\sqrt{n}) + 1$
- $T(n) = 2T\left(\frac{n}{2}\right) + n^3$
- $T(n) = 7T\left(\frac{n}{2}\right) + n^2$

2 Induction

Prove the following using induction:

$$\Sigma_{i=0}^{n} ar^i = \frac{ar^{n+1} - a}{r - 1}$$

3 MergeSort

Rewrite the MERGE procedure on page 29 in CLRS, so that is does not use sentinels, instead stopping once either array L or R has had all its elements copied back to A and then copying the remainder of the other array back into A.

4 Sorting

Suppose you are given an array A containing n sorted elements followed by $\lg n$ unsorted elements (assume $\lg n$ is an integer). Thus, the entire array has $N = n + \lg n$ elements.

Can the entire array be sorted in $O(n)$ time? If the answer is yes, give a detailed outline of an algorithm. Also, give the running time analysis of your algorithm.

5 Inversions

An inversion if two elements in an array A is a pair of indices $i < j$ such that $A(i) > A(j)$. Determine the number of inversions in a backward-sorted (decreasing order) array A with unique elements.