Legal Notice

The Zoom session for this class will be recorded and made available asynchronously on Canvas to registered students.
Announcements

1. HW 3 is due next Tuesday.

2. HW 4 is online, due before class in 1.5 weeks, November 3.
Last time: Hash functions

This time: Hash-based MACs, authenticated encryption
Constructing a MAC from a hash function

Recall:

- Collision-resistant hash function: Unkeyed function $H : \{0, 1\}^* \rightarrow \{0, 1\}^n$ hard to find inputs mapping to same output.

- MAC: Keyed function $\text{Mac}_k(m) = t$, hard for adversary to construct valid $(m, t)$ pair.

Hash function alone w/o MAC: anyone can forge $(m, H(m))$

No secrets.
Candidate MAC constructions

- \( \text{Mac}(k, m) = H(k\|m) \)

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- \( \text{Mac}(k_1, k_2, m) = H(k_2\|H(k_1\|m)) \)
Candidate MAC constructions

- \( \text{Mac}(k, m) = H(k||m) \)

  **Insecure.** Vulnerable to length extension attacks for Merkle-Damgård functions. Secure for SHA3 sponge.

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  Secure, similar to HMAC.
Length extension attacks

Recall the Merkle-Damgård construction:

\[ \hat{m}_k = m_k \| \text{pad} \| \text{len}(m) \]

The final output is equivalent to an intermediate state for \( H(m \| \text{pad} \| ...) \).
Length extension attacks

**Input:** Bad MAC: \((m, H(k||m))\)

**Attack:** Forge valid bad MAC: \((m||pad||m', H(k||m||pad||m'))\)

In general, we can construct the hash \(H(m||pad||m_{new})\) for any \(m_{new}\) from only \(H(m)\) even if we don’t know \(m\).

Just need to know (or guess) \(\text{len}(m)\) to compute padding.
HMAC: A PRF for Merkle-Damgård functions

\[ F_k(m) = H(k \oplus \text{opad} \parallel H(k \oplus \text{ipad}) \parallel m) \]

\[
\begin{align*}
\text{ipad} &= 0x36 \\
\text{opad} &= 0x5C
\end{align*}
\]

Under the heuristic assumption that \( k \oplus \text{opad} \) and \( k \oplus \text{ipad} \) are “independent” keys, this is a secure PRF.

HMAC is standardized and HMAC-SHA256 is a good choice. Historically HMAC-SHA1 was also common.

\[ H(k \parallel m) \] is a secure MAC for SHA3.
Key derivation

**Problem:** How do we get symmetric keys?

**Input:** Some data that we want to use to generate a key.
- A password
- A bunch of nonuniform random inputs from the environment
- The result of a public-key agreement (coming soon!)

**Desired output:** Uniform AES or MAC keys of the right length.

**Solutions that work in practice:**
- $H(data)$
- $HMAC_0(data)$ (better for Merkle-Damgård functions)
Subkey derivation

For a real protocol, we likely need several keys: encryption keys for each direction, MAC keys.

Once we have derived a master key $mk$ using a hash function, we can use a PRF to derive subkeys.

Examples:

- $k_{mac} = F_{mk}("MAC-KEY")$
- $k_{AB} = F_{mk}("AB-KEY")$ for Alice $\rightarrow$ Bob encryption
- $k_{BA} = F_{mk}("BA-KEY")$ for Bob $\rightarrow$ Alice encryption

If $F$ is a secure PRF, then these behave like independent keys.

HMAC is often used for this in practice.
HKDF

Standardized HMAC-based key derivation function.

**Input:** Secret $s$, optional salt $salt$

**Output:** $L$ bytes of output

**Algorithm:**
Use a HMAC function with output length $\ell$.

1. $t = HMAC_{salt}(s)$
2. $z_0 = $ empty string.
3. for $i$ from 1 to $\lceil L/\ell \rceil$:
   $$z_i = HMAC_t(z_{i-1} || i)$$
4. Output $L$ bytes of $z_1 || \ldots$
Chosen ciphertext attacks

\[ A \xrightarrow{m_0, m_1} C \xrightarrow{C \leftarrow \text{Enc}(m_2)} \]

Oracle access to \( \text{Enc}(\cdot), \text{Dec}(\cdot) \)

\[ b' \xrightarrow{b = b'} A \text{ succeeds} \]

**Definition**

\((\text{Enc}, \text{Dec})\) is CCA-secure if \(\forall\) efficient adversaries \(A\),

\[ \Pr[A \text{ succeeds}] \leq 1/2 + \epsilon \]

**IND-CCA1:** Non-adaptive: Decryption oracle only queried prior to \(c\)

**IND-CCA2:** Adaptive: May make further calls to decryption oracle
Ciphertext Integrity

A wins if $c$ is a valid ciphertext and not queried.

**Definition**

(Enc, Dec) provides ciphertext integrity if $\Pr[A \text{ succeeds}] = \text{negligible}$. 
Authenticated Encryption

**Definition**

(Enc, Dec) provides authenticated encryption if it is CPA-secure and provides ciphertext integrity.

**Theorem**

If (Enc, Dec) provides authenticated encryption then it is CCA-secure.
Constructing Authenticated Encryption

Encrypt-then-MAC

- Encryption: $c = \text{Enc}_{k_e}(m)$  $t = \text{Mac}_{k_m}(c)$  output $(c, t)$
- Decryption: Input $(c, t)$.
  If $\text{Verify}_{k_m}(c, t) = \text{reject}$ then output reject
  else output $\text{Dec}_{k_e}(c)$.

Theorem

Encrypt-then-MAC is CCA secure.

Common implementation mistakes:

- Using the same key for encryption and MAC
- Only MACing part of the ciphertext. (e.g. omitting the IV or the data used to derive a deterministic IV)
- Outputting some plaintext before verifying integrity
MAC then Encrypt is not CCA secure

MAC-then-encrypt

- Encryption: $t = \text{Mac}_{k_m}(m)$, $c = \text{Enc}_{k_e}(m||t)$, output $c$
- Decryption: Input $c$. Compute $\text{Dec}_{k_e}(c) = (m||t)$
  If $\text{Verify}_{k_m}(m, t) = \text{reject}$ then output reject
  else output $m$.

MAC-then-encrypt can fail to be secure even with CPA-secure Enc
and secure MAC.

SSL 3.0 vulnerable to POODLE attack.
POODLE Attack Setup

Victim is a web browser.

Victim visits evil.com.

evil.com contains Javascript causing victim to make cookie-bearing request to bank.com.

Man-in-the-middle attacker intercepts encrypted traffic between victim and bank.com and modifies ciphertext, using bank.com as a decryption oracle.
POODLE Attack Idea

SSL 3.0 uses MAC-then-encrypt with CBC mode.

\[ c = \text{Enc}(\text{message} \ || \ \text{MAC tag} \ || \ \text{pad}) \]

To pad \( p \) bytes, append \( p - 1 \) arbitrary bytes and then byte \( p - 1 \). (For 0 bytes, append dummy block of 15 bytes ending in 15.)

If adversary intercepts block

\[ c = c[0] \hspace{1cm} c[1] \hspace{1cm} \cdots \hspace{1cm} c[\ell - 1] \hspace{1cm} c[\ell] \]

- IV
- encryption of \( m \)
- encrypted tag
- encrypted pad

Then they query decryption oracle with

\[ \hat{c} := c[0] \hspace{1cm} c[1] \hspace{1cm} \cdots \hspace{1cm} c[\ell - 1] \hspace{1cm} c[1] \]

If last byte is 15, decryption valid, otherwise likely reject
\[ \implies \text{learn byte of } m. \ (\text{Same logic as your homework.}) \]
Authenticated encryption in practice

Fine solution: Use AES-GCM mode.

- TLS 1.3 uses authenticated encryption modes correctly.
- Older versions of TLS use MAC-then-encrypt.
- SSHv2 uses Encrypt-and-MAC. This is not generally secure but is secure for SSH’s cipher choices.
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