Practice Questions for Midterm 1

Question 1
Consider the following truth table:

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>r</th>
<th>output</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
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<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
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<td>T</td>
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<td>F</td>
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<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

Write a proposition in CNF and DNF forms

Question 2
Please negate the following propositions, and simplify them with De-Morgan's Law

\[ p \rightarrow q \]
\[ (p \rightarrow q) \lor (q \land (r \rightarrow p)) \]
Question 3
Please convert the following bit string under different representations into base 10
1101 1010 0010 0111
As if the string is unsigned: __________
As if the string is in signed magnitude: __________
As if the string is in two's complement: __________

11010111010000
As if the string is unsigned: __________
As if the string is in signed magnitude: __________
As if the string is in two's complement: __________
Can you also explain pros and cons of the above representations?

Question 4
Assume we have two unsigned binary numbers in fixed width 4 such that:
x = (x_3, x_2, x_1, x_0)_{2,4} and y = (y_3, y_2, y_1, y_0)_{2,4}
Please give a formula to compute unsigned binary number, z = (z_3, z_2, z_1, z_0)_{2,4}, with fixed width 4 where z = x + y:

z_3=_______________________
z_2=_______________________
z_1=_______________________
z_0=_______________________

Question 5
Please convert the following numbers into fixed width representations:

(27)_{10}
unsigned binary fixed width 6 (_________________________ )_{2,6}
signed magnitude binary width 6 (_________________________ )_{2,6}
two's complement width 6 (_________________________ )_{2,6}
(1F6C)_{16}
unsigned binary fixed width 16 ( )_{2,16}
signed magnitude binary width 16 ( )_{2,16}
two's complement width 16 ( )_{2,16}

(712)_{8}
unsigned binary fixed width 10 ( )_{2,10}
signed magnitude binary width 10 ( )_{2,10}
two's complement width 10 ( )_{2,10}

Question 6
Let the domain D = \mathbb{Z} , translate the following sentence to a proposition

"Every odd number can be divided by some positive number"

(1) Define 2 predicates and write the proposition
(2) Write the negation
(3) Translate the negation back into English sentence

Question 7
(1) Negate the following proposition

\forall x_1 \in \mathbb{R}, \exists x_2 \in \mathbb{R}, (p \rightarrow q) \lor (P(x_1) \land P(x_2))

(2) Define a predicate P(x) such that the following statement is true

\exists x \in \mathbb{Z}, P(x) \land (x = 5)