For $b$ an integer greater than 1 and $n$ a positive integer, the **base $b$ expansion of $n$** is $(a_{k-1} \cdots a_1a_0)_b$ where $k$ is a positive integer, $a_0, a_1, \ldots, a_{k-1}$ are nonnegative integers less than $b$, $a_{k-1} \neq 0$, and $n = a_{k-1}b^{k-1} + \cdots + a_1b + a_0$.

For $b$ an integer greater than 1, $w$ a positive integer, and $n$ a nonnegative integer with $n < b^w$, the **base $b$ fixed-width $w$ expansion of $n$** is $(a_{w-1} \cdots a_1a_0)_{b,w}$ where $a_0, a_1, \ldots, a_{w-1}$ are nonnegative integers less than $b$ and $n = a_{w-1}b^{w-1} + \cdots + a_1b + a_0$.

**Representing negative integers in binary**: Fix a positive integer width for the representation $w$, $w > 1$.

<table>
<thead>
<tr>
<th>To represent a positive integer $n$</th>
<th>To represent a negative integer $-n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[0a_{w-2} \cdots a_0]<em>{s,w}$, where $n = (a</em>{w-2} \cdots a_0)_{2,w-1}$</td>
<td>$[1a_{w-2} \cdots a_0]<em>{s,w}$, where $n = (a</em>{w-2} \cdots a_0)_{2,w-1}$</td>
</tr>
<tr>
<td>Example $n = 17$, $w = 7$:</td>
<td>Example $-n = -17$, $w = 7$:</td>
</tr>
<tr>
<td>$[0a_{w-2} \cdots a_0]<em>{2c,w}$, where $n = (a</em>{w-2} \cdots a_0)_{2,w-1}$</td>
<td>$[1a_{w-2} \cdots a_0]<em>{2c,w}$, where $2^{w-1} - n = (a</em>{w-2} \cdots a_0)_{2,w-1}$</td>
</tr>
<tr>
<td>Example $n = 17$, $w = 7$:</td>
<td>Example $-n = -17$, $w = 7$:</td>
</tr>
<tr>
<td>$[0a_{w-2} \cdots a_0]<em>{1c,w}$, where $n = (a</em>{w-2} \cdots a_0)_{2,w-1}$</td>
<td>$[1\overline{a}<em>{w-2} \cdots \overline{a}<em>0]</em>{1c,w}$, where $n = (a</em>{w-2} \cdots a_0)_{2,w-1}$ and we define $\overline{0} = 1$ and $\overline{1} = 0$.</td>
</tr>
<tr>
<td>Example $n = 17$, $w = 7$:</td>
<td>Example $-n = -17$, $w = 7$:</td>
</tr>
</tbody>
</table>

**Representing 0:**
**Fixed-width addition:** adding one bit at time, using the usual column-by-column and carry arithmetic, and dropping the carry from the leftmost column so the result is the same width as the summands. *Does this give the right value for the sum?*

\[
\begin{array}{c}
(1\ 1\ 0\ 1\ 0\ 0)_{2,6} \\
+ (0\ 0\ 0\ 1\ 0\ 1)_{2,6}
\end{array}
\quad
\begin{array}{c}
[1\ 1\ 0\ 1\ 0\ 0]_{8,6} \\
+ [0\ 0\ 0\ 1\ 0\ 1]_{8,6}
\end{array}
\quad
\begin{array}{c}
[1\ 1\ 0\ 1\ 0\ 0]_{2c,6} \\
+ [0\ 0\ 0\ 1\ 0\ 1]_{2c,6}
\end{array}
\]

**Extra example**

\[
\begin{array}{c}
(1\ 1\ 0\ 1\ 0\ 0)_{2,6} \\
\times (0\ 0\ 0\ 1\ 0\ 1)_{2,6}
\end{array}
\quad
\begin{array}{c}
[1\ 1\ 0\ 1\ 0\ 0]_{8,6} \\
\times [0\ 0\ 0\ 1\ 0\ 1]_{8,6}
\end{array}
\quad
\begin{array}{c}
[1\ 1\ 0\ 1\ 0\ 0]_{2c,6} \\
\times [0\ 0\ 0\ 1\ 0\ 1]_{2c,6}
\end{array}
\]

**Example digital circuit:**

- Output when \(x = 1, y = 0, z = 0, w = 1\) is ____
- Output when \(x = 1, y = 1, z = 1, w = 1\) is ____
- Output when \(x = 0, y = 0, z = 0, w = 1\) is ____