Stereo
(Part 2)

Computer Vision I
CSE 252A
Lecture 9
Announcements

• Homework 3 is due Nov 5, 11:59 PM
• Reading:
  – Chapter 7: Stereopsis
Stereo Vision Outline

• Offline: Calibrate cameras & determine “epipolar geometry”
• Online
  1. Acquire stereo images
  2. Rectify images to convenient epipolar geometry
  3. Establish correspondence
  4. Estimate depth
Determine Epipolar Geometry

- **Essential matrix**
  - Linear estimate
    - From point correspondences in normalized coordinates
    - Enforce constraints
      - Singular values are (1, 1, 0)

- **Fundamental matrix**
  - Linear estimate (with data normalization)
    - Data normalize point correspondences
    - Estimate data normalized fundamental matrix
    - Enforce constraints
      - Smallest singular value is zero
    - Data denormalize data normalized, constraint enforced fundamental matrix
Rectification
Given a pair of images, transform both images so that epipolar lines are scan lines.
Rectification

Under perspective projection, the mapping from a plane to a plane is given by a projective transformation (aka homography).

\[
\begin{bmatrix}
  x_L \\
  y_L \\
  w_L
\end{bmatrix}
= H_L
\begin{bmatrix}
  u_L \\
  v_L \\
  1
\end{bmatrix}
\]
Rectification

Under perspective projection, the mapping from a plane to a plane is given by a projective transformation (aka homography).

\[
\begin{bmatrix}
  x_L \\
y_L \\
w_L
\end{bmatrix} = H_L \begin{bmatrix}
u_L \\
v_L \\
1
\end{bmatrix}
\]

\[
\begin{bmatrix}
x_r \\
y_r \\
w_r
\end{bmatrix} = H_R \begin{bmatrix}
u_r \\
v_r \\
1
\end{bmatrix}
\]

Two images – Two homographies
Epipolar Rectification

- Create pair of virtual cameras
  - Virtual cameras have the same camera centers as real cameras
  - Both virtual cameras have the same:
    - Camera rotation matrix $R$
    - Camera calibration matrix $K$
- Rectification transformation matrices
  $$H = K_{\text{virtual}} R_{\text{virtual}} R_{\text{real}}^T K_{\text{real}}^{-1}$$
Image pair rectification

Simplify stereo matching by warping the images

Apply projective transformation so that epipolar lines correspond to horizontal scanlines

H should map epipole e to (1,0,0), a point at infinity
H should minimize image distortion

Note that rectified images usually not rectangular
See Text for complete method

\[
\begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix} = He
\]
Rectification
Given a pair of images, transform both images so that epipolar lines are scan lines.

Input Images
Rectification
Given a pair of images, transform both images so that epipolar lines are scan lines.

Rectified Images
See Section 7.2.1 for specific method
Rectification
Rectification

• Epipolar lines
Rectification
Polar Rectification

Homography-based Rectification

Polar Rectification

Alternative epipolar rectification method that minimizes pixel distortion
Polar Rectification

Epipoles are in images (white dot on ball)

Homography-based rectification is not possible
Features on same epipolar line
Dense Correspondence:  
A Photometric constraint

• A point in the world has same intensity in both images (Constant Brightness Constraint)
  – This applies when scene is Lambertian and tangent plane is parallel to image plane
Mobi: Stereo-based navigation
Epipolar correspondence
Symbolic Map
Using epipolar & constant Brightness constraints for stereo matching

For each epipolar line
  For each pixel in the left image
    • compare with every pixel on same epipolar line in right image
      • pick pixel with minimum match cost
        • This will never work, so:

        match windows

(Seitz)
Finding Correspondences

$W(p_1)$

$W(p_r)$
Next Lecture

• Early vision: multiple images
  – Stereo

• Reading:
  – Chapter 7: Stereopsis