Edge Detection and Corner Detection

Computer Vision I
CSE 252A
Lecture 7
Announcements

• Homework 2 is due Oct 22, 11:59 PM
• Homework 3 will be assigned on Oct 22
• Reading:
  – Chapter 5: Local Image Features
Edges
Corners
Edges

What is an edge?

A discontinuity in image intensity.

Physical causes of edges

1. Object boundaries
2. Surface normal discontinuities
3. Reflectance (albedo) discontinuities
4. Lighting discontinuities (shadow boundaries)
Object Boundaries
Surface normal discontinuities
Boundaries of materials properties
Boundaries of lighting
Profiles of image intensity edges

Step Edges

Roof Edge

Line Edges
Noisy Step Edge

- Derivative is high everywhere.
- Must smooth before taking gradient.
Edge is Where Change Occurs: 1-D

• Change is measured by derivative in 1D

- Ideal Edge: Biggest change, derivative has maximum magnitude
- Smoothed Edge
- First Derivative
- Second Derivative

• Biggest change, derivative has maximum magnitude
• Or 2nd derivative is zero.
Numerical Derivatives

Take Taylor series expansion of \( f(x) \) about \( x_0 \)

\[
f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{1}{2} f''(x_0)(x-x_0)^2 + \ldots
\]

Consider samples taken at increments of \( h \) and first two terms of the expansion, we have

\[
f(x_0+h) = f(x_0) + f'(x_0)h + \frac{1}{2} f''(x_0)h^2
\]

\[
f(x_0-h) = f(x_0) - f'(x_0)h + \frac{1}{2} f''(x_0)h^2
\]

Subtracting and adding \( f(x_0+h) \) and \( f(x_0-h) \) respectively yields

\[
f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h}
\]

\[
f''(x_0) = \frac{f(x_0+h) - 2f(x_0) + f(x_0-h)}{h^2}
\]

Convolve with

First Derivative: \([-1/2h \ 0 \ 1/2h]\)

Second Derivative: \([1/h^2 \ -2/h^2 \ 1/h^2]\)
Numerical Derivatives

Convolution kernel
First Derivative: \([-1/2h\ 0\ 1/2h]\)
Second Derivative: \([1/h^2\ -2/h^2\ 1/h^2]\)

• With images, units of h is pixels, so h=1
  – First derivative: \([-1/2\ 0\ 1/2]\)
  – Second derivative: \([1\ -2\ 1]\)

• When computing derivatives in the x and y directions, use these convolution kernels:

\[
\frac{d}{dx} = \begin{bmatrix} -1/2 & 0 & 1/2 \end{bmatrix} \\
\frac{d}{dy} = \begin{bmatrix} -1/2 \\ 0 \\ 1/2 \end{bmatrix}
\]
Implementing 1-D Edge Detection

1. Filter out noise: convolve with Gaussian

2. Take a derivative: convolve with \([-1/2 0 1/2]\)
   - We can combine 1 and 2.

3. Find the peak: Two issues:
   - Should be a local maximum.
   - Should be sufficiently high.
2D Edge Detection

1. Filter out noise
   - Use a 2D Gaussian Filter.

2. Take a derivative
   - Compute the magnitude of the gradient:

   \[ \nabla J = (J_x, J_y) = \left( \frac{\partial J}{\partial x}, \frac{\partial J}{\partial y} \right) \]

   is the gradient

   \[ \| \nabla J \| = \sqrt{J_x^2 + J_y^2} \]

   is the magnitude of the gradient

   \[ \tan^{-1}(J_y, J_x) \]

   the direction of the gradient
Smoothing and Differentiation

• Need two derivatives, in x and y direction.
• Filter with Gaussian and then compute Gradient, OR
• Use a derivative of Gaussian filter
  • because differentiation is convolution, and convolution is associative
Directional Derivatives

\[
\begin{align*}
\frac{\partial G_\sigma}{\partial x} \\
\frac{\partial G_\sigma}{\partial y}
\end{align*}
\]

\[
\cos \theta \frac{\partial G_\sigma}{\partial x} + \sin \theta \frac{\partial G_\sigma}{\partial y}
\]
Gradient

- Given a function \( f(x,y) \) -- e.g., intensity is \( f \)

- Gradient equation: \( \nabla f = \left[ \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right] \)

- Represents direction of most rapid change in intensity

- Gradient direction: \( \theta = \tan^{-1} \left( \frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right) \)

- The edge strength is given by the gradient magnitude

\[
\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}
\]
Finding derivatives

Is this $dI/dx$ or $dI/dy$?
There are three major issues:

1. The gradient magnitude at different scales is different; which scale should we choose?
2. The gradient magnitude is large along a thick trail; how do we identify the significant points?
3. How do we link the relevant points up into curves?
There is ALWAYS a tradeoff between smoothing and good edge localization!

- Image with Edge (No Noise)
- Edge Location
- Image + Noise
- Derivatives detect edge and noise
- Smoothed derivative removes noise, but blurs edge
The scale of the smoothing filter affects derivative estimates
We wish to mark points along the curve where the magnitude is biggest. We can do this by looking for a maximum along a slice normal to the curve (non-maximum suppression). These points should form a curve. There are then two algorithmic issues: which point is the maximum, and where is the next point on the curve?
Non-maximum suppression

Using normal at q, find two points p and r on adjacent rows (or columns)

q is a maximum if $|\nabla J(q)|$ is larger than $|\nabla J(p)|$ and $|\nabla J(r)|$

Interpolate to get values
Before Non-max Suppression

Gradient magnitude (x4 for visualization)
After non-max suppression

Gradient magnitude (x4 for visualization)
Non-maximum suppression
Predicting the next edge point

- The marked point is an edge point.
- From edge tangent (normal to gradient), predict next point along edge curve (here either r or s)
- Link together to create edge curve
Input image
Single Threshold

- When threshold is too high, important edges may be missed or be broken
- When threshold is too low, many extraneous edges, but non missed
- Hysteresis thresholding: Best of both
Hysteresis Thresholding

- Start tracking an edge chain at pixel location that is local maximum of gradient magnitude where gradient magnitude > $\tau_{\text{high}}$.
- Follow edge in direction orthogonal to gradient.
- Stop when gradient magnitude < $\tau_{\text{low}}$.
  - i.e., use a high threshold to start edge curves and a low threshold to continue them.
Single Threshold

$T = 15$

$T = 5$

Hysteresis

$T_h = 15 \quad T_l = 5$

Hysteresis thresholding
Canny Edge Detection Algorithm

1. Three parameters $\sigma$, $\tau_{\text{high}}$, $\tau_{\text{low}}$
2. Filter with symmetric Gaussian of width $\sigma$
3. Computer gradient, magnitude, direction
4. Non-maximal supression
5. Hysteresis thresholding using $\tau_{\text{high}}$, $\tau_{\text{low}}$
fine scale, high threshold
coarse scale, high high threshold
coarse scale, low high threshold
Why is Canny so Dominant

- Widely used for 30 years.
- Theory is nice
- Details are good
  - Magnitude of gradient,
  - Non-max supression
  - Hysteresis thresholding
- Most subsequent detectors weren’t much better until learning-based detectors came along
- Code was distributed
Learning-based detectors: Not edges, but boundaries

- Brightness
- Color
- Texture
- Subjective contours
- Grouping
- Multiscale
Boundary detection

- Precision is the fraction of detections that are true positives rather than false positives, while recall is the fraction of true positives that are detected rather than missed.

Learned Edge Detectors


• Xie, Saining, and Zhuowen Tu. "Proceedings of the IEEE international conference on computer vision. 2015.

HED Performance

Xie and Tu. "Holistically-nested edge detection." ICCV 2015
Corner Detection
Feature extraction: Corners
Why extract features?

- Motivation: panorama stitching
  - We have two images – how do we combine them?
Why extract features?

• Motivation: panorama stitching
  – We have two images – how do we combine them?

Step 1: extract features
Step 2: match features
Why extract features?

• Motivation: panorama stitching
  – We have two images – how do we combine them?

Step 1: extract features
Step 2: match features
Step 3: align images
Corners contain more info than lines.

- A point on a line is hard to match.
Corners contain more info than lines.

- A corner is easier to match
The Basic Idea

• We should easily recognize the point by looking through a small window
• Shifting a window in any direction should give a large change in intensity

“flat” region: no change in all directions

“edge”: no change along the edge direction

“corner”: significant change in all directions

Source: A. Efros
CSE 252A, Fall 2019
Finding Corners

Intuition:

- Right at corner, gradient is ill-defined.
- Near corner, gradient has two different values.
Distribution of gradients for different image patches

- **Flat region**
- **Edge**
- **Corner**

Derivative distribution of different regions
Formula for Finding Corners

Shi-Tomasi Detector

For each image location \((x,y)\), we create a matrix \(C(x,y)\):

\[
C(x, y) = \begin{bmatrix}
\sum \sum \sum I_x^2 & \sum \sum \sum I_x I_y \\
\sum \sum \sum I_x I_y & \sum \sum \sum I_y^2
\end{bmatrix}
\]

Matrix is symmetric

Sum over a small region

Gradient with respect to \(x\), times gradient with respect to \(y\)

WHY THIS?
Because \( C \) is a symmetric positive semidefinite matrix, it can be factored as:

\[
C = R^{-1} \begin{bmatrix}
\lambda_1 & 0 \\
0 & \lambda_2
\end{bmatrix} R = R^T \begin{bmatrix}
\lambda_1 & 0 \\
0 & \lambda_2
\end{bmatrix} R
\]

where \( R \) is a 2x2 rotation matrix and \( \lambda_1 \) and \( \lambda_2 \) are non-negative.

1. \( \lambda_1 \) and \( \lambda_2 \) are the Eigenvalues of \( C \).
2. The columns of \( R \) are the Eigenvectors of \( C \).
3. Eigenvalues can be found by solving the characteristic equation \( \det(C-\lambda I) = 0 \) for \( \lambda \).
What is region like if:

1. $\lambda_1 = 0$, $\lambda_2 > 0$?
2. $\lambda_2 = 0$, $\lambda_1 > 0$?
3. $\lambda_1 = 0$ and $\lambda_2 = 0$?
4. $\lambda_1 \gg 0$ and $\lambda_2 \gg 0$?
Shi-Tomasi Corner Detector

- Filter image with a Gaussian.
- Compute the gradient everywhere.
- Move window over image, and for each window location:
  1. Construct the matrix $C$ over the window.
  2. Use linear algebra to find $\lambda_1$ and $\lambda_2$.
  3. If they are both large, we have a corner.
     1. Let $e(x,y) = \min(\lambda_1(x,y), \lambda_2(x,y))$
     2. $(x,y)$ is a corner if it’s local maximum of $e(x,y)$
        and $e(x,y) > \tau$

Parameters: Gaussian std. dev, window size, threshold
Corner Detection Sample Results

Threshold=25,000

Threshold=10,000

Threshold=5,000
Next Lecture

• Early vision: multiple images
  – Stereo

• Reading:
  – Chapter 7: Stereopsis