Photometric Stereo

Computer Vision I
CSE 252A
Lecture 4
Announcements

- Homework 1 is due today by 11:59 PM
- Homework 2 will be assigned today
  - Due Tue, Oct 22, 11:59 PM
- Reading:
  - Section 2.2.4: Photometric Stereo
    - Shape from Multiple Shaded Images
Shading reveals 3-D surface geometry
Two shape-from-X methods that use shading

- **Shape-from-shading:** Use just one image to recover shape. Requires knowledge of light source direction and BRDF everywhere. Too restrictive to be useful.

- **Photometric stereo:** Single viewpoint, multiple images under different lighting.
Photometric Stereo Rigs:
One viewpoint, changing lighting
An example of photometric stereo

Surface (albedo texture map) + surface normals

Albedo
Photometric stereo

Single viewpoint, multiple images under different lighting.
1. Arbitrary known BRDF, known lighting
2. Lambertian BRDF, known lighting
3. Lambertian BRDF, unknown lighting
1. Photometric Stereo: General BRDF and Reflectance Map
BRDF

- Bi-directional Reflectance Distribution Function
  \[ \rho(\theta_{\text{in}}, \phi_{\text{in}} ; \theta_{\text{out}}, \phi_{\text{out}}) \]

- Function of
  - Incoming light direction:
    \[ \theta_{\text{in}}, \phi_{\text{in}} \]
  - Outgoing light direction:
    \[ \theta_{\text{out}}, \phi_{\text{out}} \]

- Ratio of incident irradiance to emitted radiance
Coordinate system

Surface: \( s(x,y) = (x, y, f(x,y)) \)

Tangent vectors:
\[
\frac{\partial s(x,y)}{\partial x} = \left( 1, 0, \frac{\partial f}{\partial x} \right)
\]
\[
\frac{\partial s(x,y)}{\partial y} = \left( 0, 1, \frac{\partial f}{\partial y} \right)
\]

Normal vector
\[
n = \frac{\partial s}{\partial x} \times \frac{\partial s}{\partial y} = \left( -\frac{\partial f}{\partial x}, -\frac{\partial f}{\partial y}, 1 \right)
\]
Gradient Space : (p,q)

\[ p = \frac{\partial f}{\partial x}, \quad q = \frac{\partial f}{\partial y} \]

Normal vector

\[
\mathbf{n} = \frac{\partial s}{\partial x} \times \frac{\partial s}{\partial y} = \left( -\frac{\partial f}{\partial x}, -\frac{\partial f}{\partial y}, 1 \right)^T
\]

\[
\hat{\mathbf{n}} = \frac{1}{\sqrt{p^2 + q^2 + 1}} \left( -p, -q, 1 \right)^T
\]
Image Formation

For a given point A on the surface \( a \), the image irradiance \( E(x,y) \) is a function of

1. The BRDF at A
2. The surface normal at A
3. The direction of the light source
Let the BRDF be the same at all points on the surface, and let the light direction $s$ be a constant.

1. Then image irradiance is a function of only the direction of the surface normal.
2. In gradient space, we have $E(p,q)$. 
Example Reflectance Map: Lambertian surface

$E(p,q)$

For lighting from front
LAMBERTIAN REFLECTANCE MAP

\[ E = L\rho \frac{1 + pp_s + qq_s}{\sqrt{1 + p^2 + q^2} \sqrt{1 + p_s^2 + q_s^2}} \]

\[ p_s = -2 \quad q_s = -1 \]

Light Source Direction, expressed in gradient space.
Reflectance Map of Lambertian Surface

What does the intensity (irradiance) of one pixel in one image tell us?

It constrains the surface normal projecting to that point to a curve

E.g., Normal lies on this curve

Light source direction in (p,q) space

i=0.5
Two Light Sources

Two reflectance maps

A third image would disambiguate match
Three Source Photometric stereo: Step 1

Offline:
Using source directions & BRDF, construct reflectance map for each light source direction. $R_1(p,q), R_2(p,q), R_3(p,q)$

Online:
1. Acquire three images with known light source directions. $E_1(x,y), E_2(x,y), E_3(x,y)$
2. For each pixel location $(x,y)$, find $(p,q)$ as the intersection of the three curves
   - $R_1(p,q)=E_1(x,y)$
   - $R_2(p,q)=E_2(x,y)$
   - $R_3(p,q)=E_3(x,y)$
3. This is the surface normal at pixel $(x,y)$. Over image, the normal field is estimated
Normal Field
Plastic Baby Doll: Normal Field
Next step:
Go from normal field to surface
Recovering the surface $f(x,y)$

Many methods: Simplest approach

1. From estimate $\mathbf{n} = (n_x, n_y, n_z)$, $p = -n_x/n_z$, $q = -n_y/n_z$
2. Integrate $p = df/dx$ along a row $(x,0)$ to get $f(x,0)$
3. Then integrate $q = df/dy$ along each column starting with value of the first row
What might go wrong?

• Height \( z(x,y) \) is obtained by integration along a curve from \( (x_0, y_0) \).

\[
z(x, y) = z(x_0, y_0) + \int_{(x_0, y_0)}^{(x, y)} (pdx + qdy)
\]

• If one integrates the derivative field along any closed curve, one expects to get back to the starting value.

• Might not happen because of noisy estimates of \((p,q)\)
What might go wrong?

Integrability. If \( f(x,y) \) is the height function, we expect that

\[
\frac{\partial}{\partial y} \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \frac{\partial f}{\partial y}
\]

In terms of estimated gradient space \((p,q)\), this means:

\[
\frac{\partial p}{\partial y} = \frac{\partial q}{\partial x}
\]

But since \( p \) and \( q \) were estimated independently at each point as intersection of curves on three reflectance maps, equality is not going to exactly hold.
Horn’s Method
[ “Robot Vision, B.K.P. Horn, 1986 ]

• Formulate estimation of surface height $z(x,y)$ from gradient field by minimizing cost functional:

$$\int \int \left( z_x - p \right)^2 + \left( z_y - q \right)^2 \, dx \, dy$$

where $(p,q)$ are estimated components of the gradient while $z_x$ and $z_y$ are partial derivatives of best fit surface

• Solved using calculus of variations – iterative updating

• $z(x,y)$ can be discrete or represented in terms of basis functions.

• Integrability is naturally satisfied.
What if the BRDF unknown

Simultaneous recovery of shape and spatially varying reflectance of a surface from photometric stereo images

[Alldrin, Zickler, Kriegman, "Photometric Stereo With Non-Parametric and Spatially-Varying Reflectance", CVPR 2008]

See also [Santo, Samejima, Sugano, Shi, Matsushita, “Deep Photometric Stereo Network,” ICCV 2017]
2. Photometric Stereo:
Lambertian Surface, Known Lighting
Lambertian Surface

At image location \((u,v)\), the intensity of a pixel \(x(u,v)\) is:

\[
e(u,v) = [a(u,v) \hat{n}(u,v)] \cdot [s_0 \hat{s}]
\]

\[
= b(u,v) \cdot s
\]

where

- \(a(u,v)\) is the albedo of the surface projecting to \((u,v)\).
- \(\hat{n}(u,v)\) is the direction of the surface normal.
- \(s_0\) is the light source intensity.
- \(\hat{s}\) is the direction to the light source.
Lambertian Photometric stereo

- If the light sources $s_1$, $s_2$, and $s_3$ are known, then we can recover $b$ from as few as three images. (Photometric Stereo: Silver 80, Woodham81).

\[
\begin{bmatrix} e_1 & e_2 & e_3 \end{bmatrix} = b^T \begin{bmatrix} s_1 & s_2 & s_3 \end{bmatrix}
\]

- i.e., we measure $e_1$, $e_2$, and $e_3$ and we know $s_1$, $s_2$, and $s_3$. We can then solve for $b$ by solving a linear system.

\[
b^T = \begin{bmatrix} e_1 & e_2 & e_3 \end{bmatrix} \begin{bmatrix} s_1 & s_2 & s_3 \end{bmatrix}^{-1}
\]

- Normal $\hat{n} = b/|b|$ and albedo $a = |b|$
What if we have more than 3 Images?

Linear Least Squares

\[
[e_1 \ e_2 \ e_3 \ldots \ e_n] = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix} = [s_1 \ s_2 \ s_3 \ldots \ s_n]
\]

Let the residual be
\[ r = e - Sb \]

Squaring this:
\[
r^2 = r^T r = (e - Sb)^T (e - Sb) = e^T e - 2b^T S^T e + b^T S^T S b
\]

\[ (r^2)_b = 0 \] - zero derivative is a necessary condition for a minimum, or
\[ -2S^T e + 2S^T S b = 0; \]

Solving for \( b \) gives
\[ b = (S^T S)^{-1} S^T e \]
Input Images
Recovered albedo
Recovered normal field
Surface recovered by integration
An example of photometric stereo

Images with known associated light sources

Albedo

Surface (from normals)

Surface (albedo texture map)
Next Lecture

• Illumination cones
  – Photometric Stereo with unknown lighting and Lambertian surfaces

• Reading:
  – What Is the Set of Images of an Object under All Possible Illumination Conditions?