Q1: Which of the following sets are countably infinite? (select all that apply.) *

- The set of positive integers {1,2,...} ✔️
- The set of all real numbers between 0 and 1 (both inclusive)
- The set of all rational numbers between -2 and 2 (both inclusive) ✔️
- The set of all languages over {0,1}
- The set of all regular languages over {0,1} ✔️
- The set of all strings over {0,1} ✔️

Feedback

The 1st option (for positive integers) comes from the definition of countability. A rational number can be represented as p/q, which is a cross product of two countable sets. A regular language is defined as a language which can be defined by a DFA having a finite number of states, which means a regular language corresponds to the number of DFAs you can construct. The set of strings over {0,1} can be bijected onto the set of integers.
Q2: Is the set of non-regular languages closed under intersection? *

- Yes, because the set of regular languages is closed under intersection.
- No, because the set of regular languages is closed under intersection.
- Yes, because there are two disjoint non-regular languages.
- No, because there are two disjoint non-regular languages

Feedback

Consider 2 languages \( L_1 = \{1^k 0 q^k, k>0\} \) and \( L_2 = \{0^k 1 0^k, k>0\} \). Clearly, both these languages are non-regular. Further, they also have no strings in common, meaning that their intersection will give us the empty language, which we know is finite!
Q3: Which of the following languages are regular (select all that apply).

- \{ 1^k w 1^k | w is a string over \{0,1\} and k \geq 1 \} ✔
- \{ uw | u, w are strings over \{0,1\} with the same number of 1s \} ✔
- \{ 1^k 0 1^k | k \geq 1 \}
- None of the above

Feedback

\{1^k w 1^k\} will be regular, even though intuitively, it looks like we need to count the number of 1s. The reason for this is that the string w is unrestricted. This means that as long as your input string starts and ends with 1, you can consider all the remaining alphabets in the input to be contained in w. Hence, this language is equivalent to a language which starts and ends with a 1.

For the 2nd language, again we have no restrictions on u and w. Hence, any string with an even number of 1s can be divided into two parts u and w such that both parts have an equal number of 1s. Hence, this language accepts all strings which have an even number of 1s.

However, for the third language, we have a 0 instead of an unrestricted string w. This means that now we will have to count the number of 1’s at the start and end of the string, on either side of the middle 0, making it non-regular.
Q4: Which of the following Regular Expressions define the language recognised by the DFA given below? (Select all that apply)

- a*ba*b(a U b)*
- (a U b)*ba*ba*
- (a U b)*b(a U b)*b(a U b)*

None of the Above

Feedback

The DFA recognises all strings which contain at least 2 b's. All the Regular Expressions given generate strings with at least 2 b's as well.
Q5: Suppose $L$ is a language, and there is a string $w$ such that $w \in L$ and we can write $w = pqr$ so that $p q^i r \in L$ for all natural numbers $i$ ($i > 0$). This implies that *

- [ ] $L$ is regular
- [ ] $L$ is nonregular
- [✓] $L$ could be either regular or nonregular
- [ ] Other: 

Feedback

The purpose of this question was to remind you that pumping lemma must be checked for $i=0$ as well (pumping down). So, unless you check for $i=0$, you cannot claim with certainty that the language is either regular or non-regular.