CSE 105, Fall 2019 - Homework 7

Due: Tuesday 12/03 5pm

Instructions Upload a single file to Gradescope for each group. All group members’ names and PIDs should be on each page of the submission. Your assignments in this class will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should always explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

Reading Sipser Sections 4.1, 4.2

Key Concepts TM, Decidability
Problem 1 (10 points)

True/False. Briefly justify your answer for each statement. (You don’t need a formal proof here.)

1. Any subset of a decidable language is recognizable.
2. Any superset of an undecidable language is uncountable.
3. There exists a language that is both countable and undecidable.
4. Let \( L \) be a regular language, and \( S_L \) be a subset of \( L \). Then \( S_L \) is turing-recognizable.
5. If \( A_{TM} \) reduces to a language \( L \), then \( L \) contains infinitely many strings.

Problem 2 (10 points)

Let \( L_1, L_2, L_3 \) be three Turing-recognizable languages over an alphabet \( \Sigma \) such that they satisfy the following conditions:

\[
L_1 \cup L_2 \cup L_3 = \Sigma^* \\
L_1 \cap L_2 \cap L_3 = \varnothing
\]

Prove that \( L_1 \) is Turing-decidable.

Problem 3 (10 points)

Prove that the language \( L_{FINITE} = \{<M>: M \text{ is a Turing Machine and } L(M) \text{ is finite}\} \) is undecidable.

Problem 4 (10 points)

Let \( L_2 = \{<M>: M \text{ is a TM and } |L(M)| = 2\} \). In other words, \( L_2 \) consists of all encodings of turing machines that accept exactly 2 strings. Show that \( L_2 \) is undecidable.

Problem 5 (10 points)

Let \( L_n = \{<M>: |M \text{ is a TM and } |L(M)| = n\} \) for \( n = 0, 1, 2, 3, \ldots \)

1. Prove that \( L_n \) reduces to \( L_{n+1} \), for every \( n \geq 0 \)
2. Prove that \( L_{n+1} \) reduces to \( L_n \), for every \( n \geq 1 \)