Instructions

Upload a single file to Gradescope for each group. All group members’ names and PIDs should be on each page of the submission. Your assignments in this class will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should always explain how you arrived at your conclusions, using mathematically sound reasoning. Whether you use formal proof techniques or write a more informal argument for why something is true, your answers should always be well-supported. Your goal should be to convince the reader that your results and methods are sound.

For questions that only ask for diagrams, justifications are not required but highly recommended. It helps to show your logic in achieving the answers and partial credit can be given if there are minor mistakes in the diagrams.

Reading Sipser Sections 1.1 - 1.3

Key Concepts Deterministic finite automata (DFA), state diagram, computation trace, accept / reject, language of an automaton, regular language, union of languages, concatenation of languages, star of a language, closure of the class of regular languages under certain operations, nondeterministic finite automata (NFA), nondeterministic computation, ε arrows, equivalence of NFAs and DFAs.
Problem 1 (10 points)

For each of the below parts, draw the minimal state diagram of the DFA that recognizes the given language.

a. $L = \text{the empty language } \emptyset \text{ with } \Sigma = \{a, b\}$

b. $L = \text{the language that accepts only the empty string } \varepsilon \text{ with } \Sigma = \{a, b\}$

c. $L = \{w \in \Sigma^* \mid w \text{ does not contain an equal number of occurrences of the substrings } 01 \text{ and } 10\} \text{ with } \Sigma = \{0, 1\}$

Problem 2 (10 points)

(a) Draw the state diagram of the DFA that recognizes the language $L = \{w \in \{0, 1\}^* \mid w \text{ contains exactly one occurrence of the substring } 01\}$

For full credit your DFA must have no more than five states.

b. Draw the state diagram of the NFA that recognizes the language $L = \{w \in \Sigma^* \mid w \text{ is a palindrome of length } 4\}$

For full credit your NFA should have no more than fifteen states and the minimal number of transitions in the diagram.
Problem 3 (10 points)

Recall, for a language \( L \subseteq \Sigma^* \) its complement is the set of strings over \( \Sigma \) not in \( L \), denoted as \( \overline{L} = \{w \in L \} \subseteq \Sigma^* \). Also, recall that set difference is defined as \( L_1 - L_2 = \{w \in L_1, w \not\in L_2 \} \)

\[
A = \{w \in \{0,1\}^* | \text{w contains 101 as a substring} \}
\]

\[
B = \{w \in \{0,1\}^* | \text{w has an even number of zeros} \}
\]

(a) Draw the state diagram of the DFA of the following language: \( \overline{A} \cup B \)

For full credit, each DFA should have no more than 8 states.

(b) Draw the state diagram of the NFA of the following languages: \( (A)^* \circ B \)

For full credit your NFA should have no more than six states and the minimal number of transitions in the diagram.

Problem 4 (10 points)

Prove that any finite language (i.e. a language with a finite number of strings) is regular.

Problem 5 (10 points)

Prove that regular languages are closed under intersection. That is, given two regular languages \( L_1 \) and \( L_2 \), prove that \( L_1 \cap L_2 \) is regular.
Problem 6 (10 points)

Given the following state diagram of an NFA over the alphabet $\Sigma = \{0, 1\}$, convert it into the state diagram of its equivalent DFA. Give an informal description in English of what language these finite automata recognize. For full credit, your DFA should have no more than 8 states.