Extensions of latent-factor models,
(and more on the Netflix prize)
Recap

1. Measuring similarity between users/items for **binary** prediction
   *Jaccard similarity*

2. Measuring similarity between users/items for **real-valued** prediction
   *cosine/Pearson similarity*

3. Dimensionality reduction for **real-valued** prediction
   *latent-factor models*
In 2006, Netflix created a dataset of 100,000,000 movie ratings. Data looked like:

\[(\text{userID}, \text{itemID}, \text{time}, \text{rating})\]

The goal was to reduce the (R)MSE at predicting ratings:

\[
\text{RMSE}(f) = \sqrt{\frac{1}{N} \sum_{u,i,t \in \text{test set}} (f(u,i,t) - r_{u,i,t})^2}
\]

Whoever first manages to reduce the RMSE by 10% versus Netflix’s solution wins $1,000,000.
Last lecture...

Let’s start with the simplest possible model:

$$f(u, i) = \alpha$$

$$\alpha = R$$

$$\text{MSE}(f) = \nu \circ (R)$$
What about the 2\textsuperscript{nd} simplest model?

\[ f(u, i) = \alpha + \beta_u + \beta_i \]

- \( \alpha \): How much does this user tend to rate things above the mean?
- \( \beta_u \): Does this item tend to receive higher ratings than others?
- \( \beta_i \): Does this item tend to receive higher ratings than others?

\[ \beta_{\text{pitch black}} = -0.1 \]
\[ \beta_{\text{julian}} = -0.2 \]
Last lecture...

What about the 2\textsuperscript{nd} simplest model?

\[ f(u, i) = \alpha + \beta_u + \beta_i \]

\[ \text{obj} = \frac{1}{N} \sum \left( \alpha + \beta_{ui} - u_{ui} \right)^2 + \lambda \left( \sum \beta_i^2 + \sum \beta_i \right) \]

\[ \frac{\partial \text{obj}}{\partial \beta_u} = \ldots \]
Iterative procedure – repeat the following updates until convergence:

\[
\begin{align*}
\alpha^{(t)} &= \frac{\sum_{u,i \in \text{train}} (R_{u,i} - (\beta_u + \beta_i))}{N_{\text{train}}} \\
\beta_u^{(t+1)} &= \frac{\sum_{i \in I_u} R_{u,i} - (\alpha + \beta_i)}{\lambda + |I_u|} \\
\beta_i^{(t+2)} &= \frac{\sum_{u \in U_i} R_{u,i} - (\alpha + \beta_u)}{\lambda + |U_i|}
\end{align*}
\]

(exercise: write down derivatives and convince yourself of these update equations!)
Last lecture...
Looks good (and actually works surprisingly well), but doesn’t solve the basic issue that we started with

\[ f(\text{user features, movie features}) = \alpha + \beta a + \beta c \]

\[ = \langle \phi(\text{user features}), \theta_{\text{user}} \rangle + \langle \phi(\text{movie features}), \theta_{\text{movie}} \rangle \]

That is, we’re still fitting a function that treats users and items independently.
Recommending things to people

How about an approach based on **dimensionality reduction**?

i.e., let’s come up with low-dimensional representations of the users and the items so as to best explain the data
Dimensionality reduction

We already have some tools that ought to help us, e.g. from week 3:

$R = \begin{pmatrix}
5 & 3 & \cdots & 1 \\
4 & 2 & & 1 \\
3 & 1 & & 3 \\
2 & 2 & & 4 \\
1 & 5 & & 2 \\
\vdots & \vdots & \ddots & \vdots \\
1 & 2 & \cdots & 1
\end{pmatrix}$

What is the best low-rank approximation of $R$ in terms of the mean-squared error?
Dimensionality reduction

We already have some tools that ought to help us, e.g. from week 3:

\[
R = \begin{pmatrix}
5 & 3 & \cdots & 1 \\
4 & 2 & 1 \\
3 & 1 & 3 \\
\vdots & \vdots & \vdots & \vdots \\
1 & 2 & \cdots & 1
\end{pmatrix}
\]

The “best” rank-K approximation (in terms of the MSE) consists of taking the eigenvectors with the highest eigenvalues.

\[
R = U \Sigma V^T
\]

(square roots of) eigenvalues of \(RR^T\)
eigenvectors of \(RR^T\)
eigenvectors of \(R^T R\)
But! Our matrix of ratings is only partially observed; and it’s really big!

\[
R = \begin{pmatrix}
5 & 3 & \cdots & . \\
4 & 2 & & 1 \\
3 & . & & 3 \\
. & 2 & & 4 \\
1 & 5 & & . \\
. & . & . & . \\
1 & 2 & \cdots & . \\
\end{pmatrix}
\]

SVD is **not defined** for partially observed matrices, and it is **not practical** for matrices with 1Mx1M+ dimensions.
Instead, let’s solve approximately using gradient descent

\[ R = \begin{pmatrix} 5 & 3 & \cdots & \cdot \\ 4 & 2 & & 1 \\ 3 & \cdot & & 3 \\ \cdot & 2 & & 4 \\ 1 & 5 & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ 1 & 2 & \cdots & \cdot \end{pmatrix} \]

Users \( R \approx UV^T \)

K-dimensional representation of each item

K-dimensional representation of each user
Let's write this as:

\[ f(u, i) = \alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i \]
Latent-factor models

Let’s write this as:

\[ f(u, i) = \alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i \]

Our optimization problem is then

\[
\arg \min_{\alpha, \beta, \gamma} \sum_{u,i} (\alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i - R_{u,i})^2 + \lambda \left[ \sum_u \beta_u^2 + \sum_i \beta_i^2 + \sum_i \|\gamma_i\|^2_2 + \sum_u \|\gamma_u\|^2_2 \right]
\]
Latent-factor models

Problem: this is certainly not convex

\[ \alpha + \beta_1 + \beta_2 \]

"jointly convex"

1. Continuous, smooth
2. Suppose \( \delta_n, \xi_i \) is local optimum

\[ \ln \left[ \frac{1}{n} \sum_{i=1}^{n} \phi(x_i) \right] \]

\( \delta_n, \xi_i \) is also an optimum
Latent-factor models

Oh well. We’ll just solve it approximately

Observation: if we know either the user or the item parameters, the problem becomes easy

\[ f(u, i) = \alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i \]

e.g. fix gamma_i – pretend we’re fitting parameters for features
Latent-factor models

$$\arg\min_{\alpha,\beta,\gamma} \sum_{u,i} (\alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i - R_{u,i})^2 + \lambda \left[ \sum_u \beta_u^2 + \sum_i \beta_i^2 + \sum_i \|\gamma_i\|^2 + \sum_u \|\gamma_u\|^2 \right]$$

$$\text{do} b = \sum_{k \in I_u} \sum_{k \in I_u} (\alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i - R_{u,i}) + 2A \gamma_{uk}$$
Latent-factor models

This gives rise to a simple (though approximate) solution

**objective:**
\[
\arg \min_{\alpha, \beta, \gamma} \sum_{u,i} (\alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i - R_{u,i})^2 + \lambda \left[ \sum_u \beta_u^2 + \sum_i \beta_i^2 + \sum_i \| \gamma_i \|_2^2 + \sum_u \| \gamma_u \|_2^2 \right]
\]

\[
= \arg \min_{\alpha, \beta, \gamma} \text{objective}(\alpha, \beta, \gamma)
\]

1) fix \( \gamma_i \). Solve \( \arg \min_{\alpha, \beta, \gamma_u} \text{objective}(\alpha, \beta, \gamma) \)

2) fix \( \gamma_u \). Solve \( \arg \min_{\alpha, \beta, \gamma_i} \text{objective}(\alpha, \beta, \gamma) \)

3,4,5... repeat until convergence

Each of these subproblems is “easy” – just regularized least-squares, like we’ve been doing since week 1. This procedure is called **alternating least squares.**
Latent-factor models

**Observation:** we went from a method which uses **only** features:

\[ f(\text{user features, movie features}) \rightarrow \text{star rating} \]

...to one which **completely ignores** them:

\[
\arg \min_{\alpha, \beta, \gamma} \sum_{u,i} (\alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i - R_{u,i})^2 + \lambda \left[ \sum_u \beta_u^2 + \sum_i \beta_i^2 + \sum_i \| \gamma_i \|_2^2 + \sum_u \| \gamma_u \|_2^2 \right]
\]
So far we’ve followed the programme below:

1. Measuring similarity between users/items for **binary** prediction (e.g. Jaccard similarity)
2. Measuring similarity between users/items for **real-valued** prediction (e.g. cosine/Pearson similarity)
3. Dimensionality reduction for **real-valued** prediction (latent-factor models)
4. **Finally** – dimensionality reduction for **binary** prediction
One-class recommendation

How can we use **dimensionality reduction** to predict **binary outcomes**?

- In weeks 1&2 we saw **regression** and **logistic regression**. These two approaches use the same type of linear function to predict real-valued and binary outputs.
- We can apply an analogous approach to binary recommendation tasks.
One-class recommendation

This is referred to as “one-class” recommendation

- In weeks 1&2 we saw regression and logistic regression. These two approaches use the same type of linear function to predict real-valued and binary outputs
- We can apply an analogous approach to binary recommendation tasks
One-class recommendation

Suppose we have binary (0/1) observations (e.g. purchases) or positive/negative feedback (thumbs-up/down)

\[
R = \begin{pmatrix}
1 & 0 & \cdots & 1 \\
0 & 0 & & 1 \\
\vdots & \ddots & \ddots & \vdots \\
1 & 0 & \cdots & 1
\end{pmatrix}
\] or

\[
\begin{pmatrix}
-1 & ? & \cdots & 1 \\
? & ? & & -1 \\
\vdots & \ddots & \ddots & \vdots \\
1 & ? & \cdots & -1
\end{pmatrix}
\]

- Purchased vs. didn’t purchase
- Liked vs. didn’t evaluate vs. didn’t like
One-class recommendation

So far, we’ve been fitting functions of the form

\[ R \sim UV^T \]

• Let’s change this so that we maximize the difference in predictions between positive and negative items

• E.g. for a user who likes an item \( i \) and dislikes an item \( j \) we want to maximize:

\[
\max \ln \sigma (\gamma_u \cdot \gamma_i - \gamma_u \cdot \gamma_j)
\]
One-class recommendation

We can think of this as maximizing the probability of correctly predicting pairwise preferences, i.e.,

\[ p(i \text{ is preferred over } j) = \sigma(\gamma_u \cdot \gamma_i - \gamma_u \cdot \gamma_j) \]

- As with logistic regression, we can now maximize the likelihood associated with such a model by gradient ascent
- In practice it isn’t feasible to consider all pairs of positive/negative items, so we proceed by stochastic gradient ascent – i.e., randomly sample a (positive, negative) pair and update the model according to the gradient w.r.t. that pair
One-class recommendation

$$\max \ln \sigma(\gamma_u \cdot \gamma_i - \gamma_u \cdot \gamma_j)$$

$$\text{obj} \leq - \ln (1 + e^{\gamma_i \cdot \gamma_j - \gamma_i \cdot \gamma_i})$$

$$\nabla \frac{\text{obj}}{\gamma_{ik}} = - \left( \gamma_k - \gamma_i \right) e^{\gamma_i \cdot \gamma_j - \gamma_i \cdot \gamma_i} \frac{\gamma_i \cdot \gamma_j - \gamma_i \cdot \gamma_i}{1 + e^{\gamma_i \cdot \gamma_j - \gamma_i \cdot \gamma_i}}$$
Recap

1. Measuring similarity between users/items for **binary** prediction
   *Jaccard similarity*
2. Measuring similarity between users/items for **real-valued** prediction
   *cosine/Pearson similarity*
3. Dimensionality reduction for **real-valued** prediction
   *latent-factor models*
4. Dimensionality reduction for **binary** prediction
   *one-class recommender systems*
Further reading:

One-class recommendation:
http://goo.gl/08Rh59

Amazon’s solution to collaborative filtering at scale:

An (expensive) textbook about recommender systems:

Cold-start recommendation (e.g.):
http://wanlab.poly.edu/recsys12/recsys/p115.pdf
Extensions of latent-factor models,
(and more on the Netflix prize!)
Extensions of latent-factor models

So far we have a model that looks like:

\[ f(u, i) = \alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i \]

How might we extend this to:

• Incorporate features about users and items
  • Handle implicit feedback
  • Change over time

See Yehuda Koren (+Bell & Volinsky)'s magazine article:
“Matrix Factorization Techniques for Recommender Systems”
IEEE Computer, 2009
Extensions of latent-factor models

1) Features about users and/or items

(simplest case) Suppose we have **binary attributes** to describe users or items

\[ A(u) = [1,0,1,1,0,0,0,0,0,1,0,1,1] \]

attribute vector for user \( u \)

- e.g. is female
- is male
- is between 18-24yo
Extensions of latent-factor models

1) Features about users and/or items

(simplest case) Suppose we have **binary attributes** to describe users or items

- Associate a **parameter vector** with each attribute
- Each vector encodes how much a particular feature “offsets” the given latent dimensions

\[
A(u) = [1,0,1,1,0,0,0,0,0,1,0,1,0,1] 
\]

attribute vector for user \( u \)

\[
y_0 = [-0.2,0.3,0.1,-0.4,0.8] 
\]

~ “how does being male impact \( \gamma_u \)”
Extensions of latent-factor models

1) Features about users and/or items

(simplest case) Suppose we have **binary attributes** to describe users or items

- Associate a **parameter vector** with each attribute
- Each vector encodes how much a particular feature “offsets” the given latent dimensions
  - Model looks like:

\[
f(u, i) = \alpha + \beta_u + \beta_i + (\gamma_u + \sum_{a \in A(u)} \rho_a) \cdot \gamma_i
\]

- Fit as usual:

\[
\arg\min_{\alpha, \beta, \gamma, \rho} \sum_{u, i \in \text{train}} \left( f(u, i) - r_{u,i} \right)^2 + \lambda \Omega(\beta, \gamma)
\]

\[\text{error} \quad \text{regularizer}\]
2) Implicit feedback

Perhaps many users will never actually rate things, but may still interact with the system, e.g. through the movies they view, or the products they purchase (but never rate)

- Adopt a similar approach – introduce a binary vector describing a user’s actions

\[ N(u) = [1, 0, 0, 0, 1, 0, \ldots, 0, 1] \]

implicit feedback vector for user \( u \)

\[ \text{e.g. } y_0 = [-0.1, 0.2, 0.3, -0.1, 0.5] \]

Clicked on “Love Actually” but didn’t watch
Extensions of latent-factor models

2) Implicit feedback

Perhaps many users will never actually rate things, but may still interact with the system, e.g. through the movies they view, or the products they purchase (but never rate)

- Adopt a similar approach – introduce a binary vector describing a user’s actions
  - Model looks like:

\[
f(u, i) = \alpha + \beta_u + \beta_i + (\gamma_u + \frac{1}{||N(u)||} \sum_{a \in N(u)} \rho_a) \cdot \gamma_i
\]

normalize by the number of actions the user performed
Extensions of latent-factor models

3) Change over time

There are a number of reasons why rating data might be subject to temporal effects...
Extensions of latent-factor models

3) Change over time

Figure from Koren: “Collaborative Filtering with Temporal Dynamics” (KDD 2009)
Extensions of latent-factor models

3) Change over time

Figure from Koren: “Collaborative Filtering with Temporal Dynamics” (KDD 2009)

People tend to give higher ratings to older movies
Extensions of latent-factor models

3) Change over time

A few temporal effects from beer reviews
Extensions of latent-factor models

3) Change over time

There are a number of reasons why rating data might be subject to temporal effects...

- Changes in the interface
- People give higher ratings to older movies (or, people who watch older movies are a biased sample)
- The community’s preferences gradually change over time
- My girlfriend starts using my Netflix account one day
- I binge watch all 144 episodes of Buffy one week and then revert to my normal behavior
- I become a “connoisseur” of a certain type of movie
- Anchoring, public perception, seasonal effects, etc.

- e.g. “Collaborative filtering with temporal dynamics” Koren, 2009
- e.g. “Sequential & temporal dynamics of online opinion” Godes & Silva, 2012
- e.g. “Temporal recommendation on graphs via long- and short-term preference fusion” Xiang et al., 2010
- e.g. “Modeling the evolution of user expertise through online reviews” McAuley & Leskovec, 2013
3) Change over time

Each definition of temporal evolution demands a slightly different model assumption (we’ll see some in more detail later tonight!) but the basic idea is the following:

1) Start with our original model:

\[ f(u, i) = \alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i \]

2) And define some of the parameters as a function of time:

\[ f(u, i, t) = \alpha + \beta_u(t) + \beta_i(t) + \gamma_u(t) \cdot \gamma_i \]

3) Add a regularizer to constrain the time-varying terms:

\[
\arg\min_{\alpha, \beta, \gamma} \sum_{u,i,t \in \text{train}} (f(u, i, t) - r_{u,i,t})^2 + \lambda_1 \Omega(\beta, \gamma) + \lambda_2 \|\gamma(t) - \gamma(t + \delta)\|
\]

parameters should change smoothly
Extensions of latent-factor models

3) Change over time

**Case study:** how do people acquire tastes for beers (and potentially for other things) over time?

Differences between “beginner” and “expert” preferences for different beer styles.
4) Missing-not-at-random

- Our decision about whether to purchase a movie (or item etc.) is a function of how we **expect** to rate it
- Even for items we’ve purchased, our decision to **enter a rating** or write a review is a function of our **rating**
- e.g. some rating distribution from a few datasets:

Figure from Marlin et al. “Collaborative Filtering and the Missing at Random Assumption” (UAI 2007)
Extensions of latent-factor models

4) Missing-not-at-random

e.g. Men’s watches:
Extensions of latent-factor models

4) Missing-not-at-random

- Our decision about whether to purchase a movie (or item etc.) is a function of how we expect to rate it.
- Even for items we’ve purchased, our decision to enter a rating or write a review is a function of our rating.
- So we can predict ratings more accurately by building models that account for these differences.
  1. Not-purchased items have a different prior on ratings than purchased ones.
  2. Purchased-but-not-rated items have a different prior on ratings than rated ones.

Figure from Marlin et al. “Collaborative Filtering and the Missing at Random Assumption” (UAI 2007)
Moral(s) of the story

How much do these extensions help?

Moral: increasing complexity helps a bit, but changing the model can help a lot.

Figure from Koren: “Collaborative Filtering with Temporal Dynamics” (KDD 2009)
Moral(s) of the story

So what actually happened with Netflix?

• The AT&T team “BellKor”, consisting of Yehuda Koren, Robert Bell, and Chris Volinsky were early leaders. Their main insight was how to effectively incorporate temporal dynamics into recommendation on Netflix.
• Before long, it was clear that no one team would build the winning solution, and Frankenstein efforts started to merge. Two frontrunners emerged, “BellKor’s Pragmatic Chaos”, and “The Ensemble”.
• The BellKor team was the first to achieve a 10% improvement in RMSE, putting the competition in “last call” mode. The winner would be decided after 30 days.
• After 30 days, performance was evaluated on the hidden part of the test set.
• Both of the frontrunning teams had the same RMSE (up to some precision) but BellKor’s team submitted their solution 20 minutes earlier and won $1,000,000

For a less rough summary, see the Wikipedia page about the Netflix prize, and the nytimes article about the competition: [http://goo.gl/WNpy7o](http://goo.gl/WNpy7o)
Afterword

- Netflix had a class-action lawsuit filed against them after somebody de-anonymized the competition data
- $1,000,000 seems to be incredibly cheap for a company the size of Netflix in terms of the amount of research that was devoted to the task, and the potential benefit to Netflix of having their recommendation algorithm improved by 10%
- Other similar competitions have emerged, such as the Heritage Health Prize ($3,000,000 to predict the length of future hospital visits)
- But... the winning solution never made it into production at Netflix – it’s a monolithic algorithm that is very expensive to update as new data comes in*

*source: a friend of mine told me and I have no actual evidence of this claim
Moral(s) of the story

Finally...

Q: Is the RMSE really the right approach? Will improving rating prediction by 10% actually improve the user experience by a significant amount?
A: Not clear. Even a solution that only changes the RMSE slightly could drastically change which items are top-ranked and ultimately suggested to the user.

Q: But... are the following recommendations actually any good?
A1: Yes, these are my favorite movies!
or A2: No! There’s no diversity, so how will I discover new content?

predicted rating

5.0 stars  5.0 stars  5.0 stars  5.0 stars  4.9 stars  4.9 stars  4.8 stars  4.8 stars
Various extensions of latent factor models:

• Incorporating features
  \textit{e.g. for cold-start recommendation}
• Implicit feedback
  \textit{e.g. when ratings aren't available, but other actions are}
• Incorporating temporal information into latent factor models
  \textit{seasonal effects, short-term “bursts”, long-term trends, etc.}
  • Missing-not-at-random
  \textit{incorporating priors about items that were not bought or rated}
• The Netflix prize
Socially regularized recommender systems
see e.g. “Recommender Systems with Social Regularization”

\[
\arg \min_{\alpha, \beta, \gamma} \sum_{u,i \in \text{train}} (f(u, i) - r_{u,i})^2 + \lambda_1 \Omega(\beta, \gamma) + \lambda_2 \sum_{u,v \in \mathcal{E}} \|\gamma_u - \gamma_v\|
\]
Further reading:

Yehuda Koren’s, Robert Bell, and Chris Volinsky’s IEEE computer article:

Paper about the “Missing-at-Random” assumption, and how to address it:

Collaborative filtering with temporal dynamics:

Recommender systems and sales diversity: