Stereo Vision II

Computer Vision I

CSE252A

Lecture 12

Announcement

• HW2 extension until Thurs 11:59PM
• HW3 to be assigned tomorrow, due Tues 11/20
Stereo Vision Outline

- Offline: Calibrate cameras & determine "epipolar geometry"
- Online
  1. Acquire stereo images
  2. Rectify images to convenient epipolar geometry
  3. Establish correspondence
  4. Estimate depth

BINOCULAR STEREO SYSTEM
Estimating Depth
2D world with 1-D image plane

Two measurements: $X_L$, $X_R$
Two unknowns: $X$, $Z$

Constants:
- Baseline: $d$
- Focal length: $f$

\[ X = \frac{df X_L}{(X_L - X_R)} \]
\[ Z = \frac{d f}{(X_L - X_R)} \]

Disparity: $(X_L - X_R)$

$X_L = f(X/Z)$
$X_R = f((X-d)/Z)$

(Adapted from Hager)
Need for correspondence

Naïve complexity: If each image is n-pixels by n-pixels, then comparing all possible combinations is $O(n^4)$ which is brutal.

For a given point in the left image, where do we look in the right image?
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Epipolar matching

- Potential matches for $p$ have to lie on the corresponding epipolar line $l'$.
- Potential matches for $p'$ have to lie on the corresponding epipolar line $l$. 
Epipolar Geometry

- Baseline: line connecting two centers of projection
- Epipoles: Two intersection points of baseline with image planes
- Epipolar Plane: Any plane that contains the baseline
- Epipolar Lines: Pair of line from intersection of an epipolar plane with the two image planes

Family of epipolar Planes

The set of epipolar planes is a family of all planes passing through the baseline and can be parameterized by the angle about baseline
Interlude:
Skew Symmetric Matrix & Cross Product

- The cross product $\mathbf{a} \times \mathbf{b}$ of two vectors $\mathbf{a}$ and $\mathbf{b}$ can be expressed a matrix vector product $[\mathbf{a}_\times] \mathbf{b}$ where $[\mathbf{a}_\times]$ is the skew symmetric matrix:

$$[\mathbf{a}_\times] = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

- A matrix $\mathbf{S}$ is skew symmetric iff $\mathbf{S} = -\mathbf{S}^T$

Epipolar Constraint: Calibrated Case

- Two pinhole cameras
- Camera $\pi'$ and $\pi$ differ by rotation $\mathbf{R}$ and translation $\mathbf{t}$
- Let each camera be “calibrated”, with focal length 1mm, origin at the camera center, and pixel coordinates in mm.

- $\mathbf{P}$ projects to $\mathbf{p}$ in Camera $\pi$ and $\mathbf{p}'$ in Camera $\pi'$, what is relation of the coordinates of $\mathbf{p}$ and $\mathbf{p}'$?
Epipolar Constraint: Calibrated Case

The vectors $O_p$, $O'O'$, and $O'p'$ are coplanar

$$\overrightarrow{O_p} \cdot (\overrightarrow{O'O} \times \overrightarrow{O'p'}) = 0 \quad \Rightarrow \quad p \cdot [t \times (Rp')] = 0,$$

with

$$\begin{cases} p = (u, v, 1)^T \\ p' = (u', v', 1)^T \end{cases}$$

**Essential Matrix**
(Longuet-Higgins, 1981)

$$p^T E p' = 0 \text{ with } E = [t_x]R$$

Two Ways to Compute Essential Matrix

First calibrate the two camera to get intrinsic parameters and pixel coordinate can be mapped to mm.

$$p^T E p' = 0 \text{ with } E = [t_x]R$$

Method 1: Take images with each camera of calibration rig with known 3D coordinates and estimate extrinsic parameters $(R, t)$ and then $E$

Method 2: The Eight Point Algorithm. Use 8 matching points in images from calibrated camera of unknown scene.
The Eight-Point Algorithm (Longuet-Higgins, 1981)
Much more on multi-view in CSE252B!!

\[ \mathbf{p}^T \mathbf{E} \mathbf{p}' = 0 \text{ with } \mathbf{E} = [\mathbf{t} \times \mathbf{R}] \]

\[
\begin{bmatrix}
E_{11} & E_{12} & E_{13} \\
E_{21} & E_{22} & E_{23} \\
E_{31} & E_{32} & E_{33}
\end{bmatrix}
\begin{bmatrix}
u' \\
u \\
1
\end{bmatrix} = 0
\]

- Set \( E_{33} \) to 1
- Use 8 points \((u_i, v_i), i=1..8\)

Solve \( E_{11} \) to \( E_{32} \) --

The Essential Matrix

Epipolar geometry example
Example: converging cameras

courtesy of Andrew Zisserman

Example: motion parallel with image plane

(simple for stereo $\rightarrow$ rectification)
courtesy of Andrew Zisserman
Example: forward motion

The Essential Matrix and Epipolar constraint

\[ \mathbf{p}^T \mathbf{E} \mathbf{p}' = 0 \text{ with } \mathbf{E} = [\mathbf{t} \times] \mathbf{R} \]

1. The epipolar constraint is homogenous in \( \mathbf{p}, \mathbf{p}' \) and \( \mathbf{E} \)
2. It is bilinear in \( \mathbf{p} \) and \( \mathbf{p}' \). E.g., for a given value of \( \mathbf{p} \), it is linear in \( \mathbf{p}' \) and vice versa

3. Given a point \( \mathbf{p}' \) in \( \pi' \), the equation of the epipolar line \( \mathbf{l} \) in \( \pi \) is
   \[ \mathbf{a}^T \mathbf{p} = 0 \]
   where \( \mathbf{a} = \mathbf{E} \mathbf{p}' \)

4. Given a point \( \mathbf{p} \) in \( \pi \), the equation of the epipolar line \( \mathbf{l}' \) in \( \pi' \) is
   \[ \mathbf{b}^T \mathbf{p}' = 0 \]
   where \( \mathbf{b} = \mathbf{E}' \mathbf{p} \)
The Essential Matrix and Epipoles

\[ p^T E p' = 0 \text{ with } E = [t_c]R \]

5. \( E e' = 0 \) and \( E^T e = 0 \)
6. The eigenvector of \( E \) corresponding to the zero eigenvalue is the epipole \( e' \)
7. The eigenvector of \( E^T \) corresponding to the zero eigenvalue is the epipole \( e \)
8. \( E \) is singular (determinant is zero & can’t be inverted)
9. \( E \) has two equal non-zero singular

Relation of calibrated and uncalibrated coordinates

\[
\begin{bmatrix}
U \\
V \\
W
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
\]

Rigid Transformation

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}
\]

\[
H = \Pi_p M_c
\]

Mapping from 3D to Image Coordinates

\[ q = H \Pi_p M_c p \]

Mapping from 3D to calibrated coordinates used with Essential Matrix

\[ p_c = \Pi_p M_c p \]

Mapping from calibrated coordinates to image coordinates

\[ q = H p_c \]

Mapping from image coordinates to calibrated coordinates

\[ p_c = H^{-1} q \]
The Fundamental Matrix

The epipolar constraint is given by: \( p^T Ep' = 0 \) with \( E = [t \times]R \)

where \( p \) and \( p' \) are calibrated homogenous in the two images.

The relationship between the calibrated coordinates \((p, p')\) and uncalibrated coordinates \((q, q')\) can be expressed as \( p = H^{-1}q \) and \( p' = H'^{-1}q' \).

Therefore, we can express the epipolar constraint as:

\[
\begin{align*}
 p^T E p' &= 0 \\
 (H^{-1}q)^T E (H'^{-1}q') &= q^T ((H^{-1})^T EH'^{-1}) q' \\
 q^T F q' &= 0
\end{align*}
\]

where \( F = (H^{-1})^T EH'^{-1} \) is called the Fundamental Matrix.
Can be solved using 8 point algorithm WITHOUT CALIBRATION

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Epipolar constraint for Uncalibrated Cameras

\[ q^T F q' = 0 \]

1. The epipolar constraint is homogenous in \( q, q' \) and \( F \)
2. It is bilinear in \( q \) and \( q' \). E.g., for a given value of \( q \), it is linear in \( q' \) and vice versa

![Diagram](image)

3. Given a point \( q' \) in \( \pi' \), the equation of the epipolar line \( l \) in \( \pi \) is \( a^T q = 0 \)

where \( a = F q' \)

4. Given a point \( p \) in \( \pi \), the equation of the epipolar line \( l' \) in \( \pi' \) is \( b^T q' = 0 \)

where \( b = F^T q \)
The Essential Matrix and Epipoles

5. \( \mathbf{F}e' = 0 \) and \( \mathbf{F}^T \mathbf{e} = 0 \)
6. The eigenvector of \( \mathbf{F} \) corresponding to the zero eigenvalue is the epipole \( e' \)
7. The eigenvector of \( \mathbf{F}^T \) corresponding to the zero eigenvalue is the epipole \( e \)
8. \( \mathbf{F} \) is singular (determinant is zero & can’t be inverted)

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- Offline: Calibrate cameras & determine
B “epipolar geometry”
- Online
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Rectification
Given a pair of images, transform both images so that epipolar lines are scan lines.

Rectification
Under perspective projection, the mapping from a plane to a plane is given by a projective transformation (aka homography).

\[
\begin{bmatrix}
    x_L \\
y_L \\
w_L
\end{bmatrix}
= H_L
\begin{bmatrix}
u_L \\
v_L \\
1
\end{bmatrix}
\Rightarrow
(u_L, v_L)
\]

\[
\begin{bmatrix}
x_L \\
y_L
\end{bmatrix}
\Rightarrow
(x_L, y_L)
\]
Rectification

Under perspective projection, the mapping from a plane to a plane is given by a projective transformation (aka homography).

\[
\begin{bmatrix}
  x_L \\
  y_L \\
  w_L 
\end{bmatrix} = H_L \begin{bmatrix}
  u_L \\
  v_L \\
  1 
\end{bmatrix}
\]

\[
\begin{bmatrix}
  x_R \\
  y_R \\
  w_R 
\end{bmatrix} = H_R \begin{bmatrix}
  u_R \\
  v_R \\
  1 
\end{bmatrix}
\]

Two images – Two homographies $H_L$, $H_R$

Image pair rectification

Simplify stereo matching by warping the images

Apply projective transformation so that epipolar lines correspond to horizontal scanlines

$H_L$ should map epipole $e$ to $(1,0,0)$, a point at infinity

$H_L$ should also minimize image distortion

Note that rectified images usually not rectangular

See Text for complete method
Rectification
Given a pair of images, transform both images so that epipolar lines are scan lines.

Input Images

Rectified Images
Rectification