1. Consider a relation $R$ with attributes $ABCD$ and a decomposition $\rho = \{AB, BC, CD\}$ for $R$. Show that $\rho$ has lossless join with respect to $F = \{B \rightarrow A, C \rightarrow B\}$.

2. Let $R$ be a relation over attributes $ABCDE$, and

   $$F = \{AB \rightarrow C, C \rightarrow E, E \rightarrow C, C \rightarrow D, AB \rightarrow E\}.$$ 

   Show that the decomposition $\rho = \{ABC, ADE, CE\}$ is dependency preserving with respect to $F$.

3. Minimize the set of fds

   $$G = \{A \rightarrow C, AB \rightarrow C, C \rightarrow DI, CD \rightarrow I, EC \rightarrow AB, EI \rightarrow C\}.$$ 

4. Suppose we have a database for an investment firm, consisting of the following attributes: $B$ (broker), $O$ (office of a broker), $I$ (investor), $S$ (stock), $Q$ (quantity of stock owned by an investor), and $D$ (dividend paid by a stock), with the following functional dependencie: $S \rightarrow D, I \rightarrow B, IS \rightarrow Q, B \rightarrow O$.

   (a) Find a key for the relation $R$ over attributes $BOSQID$.

   (b) How many keys does $R$ have? Prove your answer.

   (c) Find a lossless join decomposition of $R$ into Boyce-Codd Normal Form.

   (d) Find a decomposition of $R$ into Third Normal Form, having lossless join and preserving dependencies.

5. Let $R$ be a relation over attributes $ABCD$ and $F = \{AB \rightarrow CD, D \rightarrow C, B \rightarrow C\}$.

   (a) Find a BCNF decomposition for $R$ and $F$ that has lossless join.

   (b) Is the decomposition found in (a) dependency preserving with respect to $F$? If not, go to (c).

   (c) Find a 3NF decomposition for $R$ and $F$ that has lossless join and is dependency preserving.