1. We can’t recommend everybody the same thing (even if they all want it!)

• So far, we have an algorithm that takes “budgets” into account, so that users are shown a limited number of ads, and ads are shown to a limited number of users

• But, all of this only applies if we see all the users and all the ads in advance

• This is what’s called an offline algorithm
Bipartite matching

On Monday we looked at **matching problems** which are a flexible way to find compatible user-to-item matches, while also enforcing “budget” constraints.

![Graph showing bipartite matching](image)
2. We need to be **timely**

- But in many settings, users/queries come in one at a time, and need to be shown some (highly compatible) ads
- But we still want to satisfy the same quality and budget constraints

- So, we need **online algorithms** for ad recommendation
What is adwords?

**Adwords** allows advertisers to bid on keywords.

- This is similar to our matching setting in that advertisers have limited budgets, and we have limited space to show ads.
What is adwords?

**Adwords** allows advertisers to bid on keywords

- This is similar to our matching setting in that advertisers have limited **budgets**, and we have limited space to show ads
- **But**, it has a number of key differences:
  1. Advertisers don’t pay for impressions, but rather they pay when their ads get clicked on
  2. We don’t get to see all of the queries (keywords) in advance – **they come one-at-a-time**
What is adwords?

**Adwords** allows advertisers to bid on keywords

- We still want to match advertisers to keywords to satisfy budget constraints
- But can’t treat it as a monolithic optimization problem like we did before
- Rather, we need an **online** algorithm
What is adwords?

Suppose we’re given

- Bids that each advertiser is willing to make for each query \( f(q, a) \)
  (this is how much they’ll pay if the ad is clicked on)
  - Each is associated with a click-through rate \( ctr(q, a) \)
- Budget for each advertiser \( b(a) \) (say for a 1-week period)
- A limit on how many ads can be returned for each query
What is adwords?

And, every time we see a query

- Return at most the number of ads that can fit on a page
- And which won’t overrun the budget of the advertiser (if the ad is clicked on)

Ultimately, what we want is an algorithm that maximizes revenue – the number of ads that are clicked on, multiplied by the bids on those ads
Competitiveness ratio

What we’d like is:

the revenue should be as close as possible to what we would have obtained if we’d seen the whole problem up front
(i.e., if we didn’t have to solve it online)

We’ll define the competitive ratio as:

\[
\frac{\text{revenue of our algorithm}}{\text{revenue of an optimal algorithm}}
\]

see http://infolab.stanford.edu/~ullman/mmds/book.pdf for more detailed definition
Let’s start with a simple version of the problem...

1. One ad per query
2. Every advertiser has the same budget
3. Every ad has the same click through rate
4. All bids are either 0 or 1
   (either the advertiser wants the query, or they don’t)
Then the greedy solution is...

- Every time a new query comes in, select any advertiser who has bid on that query (who has budget remaining)
  - What is the competitive ratio of this algorithm?
Greedy solution
The balance algorithm

A better algorithm...

• Every time a new query comes in, amongst advertisers who have bid on this query, select the one with the largest remaining budget

• How would this do on the same sequence?
The balance algorithm

A better algorithm...

- Every time a new query comes in, amongst advertisers who have bid on this query, *select the one with the largest remaining budget*

- In fact, the competitive ratio of this algorithm (still with equal budgets and fixed bids) is \((1 - 1/e) \approx 0.63\)

The balance algorithm

What if bids aren’t equal?

<table>
<thead>
<tr>
<th>Bidder</th>
<th>Bid (on q)</th>
<th>Budget</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>110</td>
</tr>
<tr>
<td>B</td>
<td>10</td>
<td>100</td>
</tr>
</tbody>
</table>
The balance algorithm

What if bids aren’t equal?

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<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
We need to make two modifications

• We need to consider the bid amount when selecting the advertiser, and bias our selection toward higher bids
• We also want to use some of each advertiser’s budget (so that we don’t just ignore advertisers whose budget is small)
The balance algorithm v2

Advertiser: $A_i$

fraction of budget remaining: $f_i$

bid on query $q$: $x_i(q)$

Assign queries to whichever advertiser maximizes:

$$\Psi_i(q) = x_i(q) \cdot (1 - e^{-f_i})$$

(could multiply by click-through rate if click-through rates are not equal)
The balance algorithm v2

Properties

- This algorithm has a competitive ratio of \((1 - \frac{1}{e})\).

- In fact, there is no online algorithm for the adwords problem with a competitive ratio better than \((1 - \frac{1}{e})\).

(proof is too deep for me...)
So far we have seen…

• An **online** algorithm to match advertisers to users (really to queries) that handles both **bids** and **budgets**
  • We wanted our **online** algorithm to be as good as the **offline** algorithm would be – we measured this using the **competitive ratio**
  • Using a specific scheme that favored high bids while trying to balance the budgets of all advertisers, we achieved a ratio of \((1 - \frac{1}{e})\).
    • And no better online algorithm exists!
We haven’t seen...

• AdWords actually uses a second-price auction (the winning advertiser pays the amount that the second highest bidder bid)

• Advertisers don’t bid on specific queries, but inexact matches (‘broad matching’) – i.e., queries that include subsets, supersets, or synonyms of the keywords being bid on
Further reading:

• Mining of Massive Datasets – “The Adwords Problem”
• AdWords and Generalized On-line Matching (A. Mehta)
CSE 258 – Lecture 14
Web Mining and Recommender Systems

Bandit algorithms
So far...

1. We’ve seen algorithms to handle budgets between users (or queries) and advertisers.
2. We’ve seen an online version of these algorithms, where queries show up one at a time.
3. Next, how can we learn about which ads the user is likely to click on in the first place?
3. How can we **learn** about which ads the user is likely to click on in the first place?

- If we see the user click on a car ad once, we know that (maybe) they have an interest in cars
- So... we know they like car ads, should we keep recommending them car ads?

- **No**, they’ll become less and less likely to click it, and in the meantime we won’t learn anything new about what else the user might like
Bandit algorithms

- Sometimes we should surface car ads (which we know the user likes),
- but sometimes, we should be willing to take a risk, so as to learn what else the user might like
Setup

At each round $t$, we select an arm to pull.
We’d like to pull the arm to maximize our total reward.

**K bandits (i.e., K arms)**

<table>
<thead>
<tr>
<th>round $t$</th>
<th>$t = 1$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>reward $g_{k,t}$</td>
<td>1 0 0 1 1 0 1 0 1</td>
<td>0 0 1 1 0 1 0</td>
<td>1 1 1 0 1 1 0</td>
<td>1 0 1 0 0 0 0 0</td>
<td>0 1 0 0 1 0 0</td>
<td>0 0 0 0 1 1 0</td>
<td>0 0 1 0 0 1 0</td>
<td>0 1 1 0 0 1 1</td>
<td>1 0 1 0 0 0 0 1</td>
</tr>
</tbody>
</table>
Setup

At each round $t$, we select an arm to pull.
- We’d like to pull the arm to maximize our total reward.
- **But** – we don’t get to see the reward function!

$K$ bandits (i.e., $K$ arms)

<table>
<thead>
<tr>
<th>round $t$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
</table>

reward $g_{k,t}$
**Setup**

At each round $t$, we select an arm to pull. We’d like to pull the arm to maximize our total reward. But—we don’t get to see the reward function! All we get to see is the reward we got for the arm we picked at each round.

<table>
<thead>
<tr>
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<th>reward $g_{k,t}$</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
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<td>2</td>
<td>?</td>
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<td>?</td>
</tr>
<tr>
<td>4</td>
<td>?</td>
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<tr>
<td>5</td>
<td>0</td>
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<tr>
<td>6</td>
<td>?</td>
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<td>?</td>
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<td>8</td>
<td>?</td>
</tr>
<tr>
<td>9</td>
<td>?</td>
</tr>
</tbody>
</table>

$K$ bandits (i.e., $K$ arms)
Setup

\[ K \quad \text{: number of arms (ads)} \]
\[ n \quad \text{: number of rounds} \]
\[ g_t = (g_{1,t}, \ldots, g_{K,t}) \in [0, 1]^K \quad \text{: rewards} \]
\[ l_t \in \{1, \ldots, K\} \quad \text{: which arm we pick at each round} \]
\[ g_{l_t,t} \in [0, 1] \quad \text{: how much (0 or 1) this choice wins us} \]

want to minimize \textbf{regret:}

\[ R_n = (\max_{i=1 \ldots K} \mathbb{E} \sum_{t=1}^{n} g_{i,t}) - \mathbb{E} \sum_{t=1}^{n} g_{l_t,t} \]

reward we could have got, if we had played optimally

reward our strategy would get (in expectation)
Goal

• We need to come up with a **strategy** for selecting arms to pull (ads to show) that would maximize our expected reward

• For the moment, we’re assuming that rewards are static, i.e., that they don’t change over time
Strategy 1 – “epsilon first”

• Pull arms at random for a while to learn the distribution, then just pick the best arm
• (show random ads for a while until we learn the user’s preferences, then just show what we know they like)

\[ \epsilon \cdot n : \text{Number of steps to sample randomly} \]
\[ (1 - \epsilon) \cdot n : \text{Number of steps to choose optimally} \]
Strategy 1 – “epsilon first”

• Pull arms at random for a while to learn the distribution, then just pick the best arm
• (show random ads for a while until we learn the user’s preferences, then just show what we know they like)
Strategy 2 – “epsilon greedy”

• Select the best lever most of the time, pull a random lever some of the time
• (show random ads sometimes, and the best ad most of the time)

\[ \epsilon \quad : \text{Fraction of times to sample randomly} \]
\[ (1 - \epsilon) \quad : \text{Fraction of times to choose optimally} \]

• Empirically, worse than epsilon-first
• Still doesn’t handle context/time
Strategy 3 – “epsilon decreasing”

- Same as epsilon-greedy (Strategy 2), but epsilon decreases over time
Strategy 4 – “Adaptive epsilon greedy”

• Similar to as epsilon-decreasing (Strategy 3), but epsilon can increase and decrease over time
Extensions

• The reward function may not be **static**, i.e., it may change each round according to some process
• It could be chosen by an **adversary**
• The reward may not be [0,1] (e.g. clicked/not clicked), but instead a could be a real number (e.g. revenue), and we’d want to estimate the distribution over rewards
Extensions – **Contextual** Bandits

- There could be **context** associated with each time step
  - The query the user typed
  - What the user saw during the **previous** time step
  - What other actions the user has recently performed
  - Etc.
Applications (besides advertising)

- Clinical trials
  (assign drugs to patients, given uncertainty about the outcome of each drug)
- Resource allocation
  (assign person-power to projects, given uncertainty about the reward that different projects will result in)
- Portfolio design
  (invest in ventures, given uncertainty about which will succeed)
- Adaptive network routing
  (route packets, without knowing the delay unless you send the packet)
Questions?

Further reading:
Tutorial on Bandits:
https://sites.google.com/site/banditstutorial/
Case study – Turning down the noise
“Turning down the noise in the Blogosphere”
(By Khalid El-Arini, Gaurav Veda, Dafna Shahaf, Carlos Guestrin)

**Goals:**

1. Help to **filter** huge amounts of content, so that users see content that is **relevant** – rather than seeing popular content over and over again
2. Maximize **coverage** so that a variety of different content is recommended
3. Make recommendations that are **personalized** to each user

some slides http://www.select.cs.cmu.edu/publications/paperdir/kdd2009-elorini-veda-shahaf-guestrin.pptx
“Turning down the noise in the Blogosphere”

(By Khalid El-Arini, Gaurav Veda, Dafna Shahaf, Carlos Guestrin)

Goals:

1. Help to filter huge amounts of content, so that users see content that is relevant – rather than seeing popular content over and over again
2. Maximize coverage so that a variety of different content is recommended
3. Make recommendations that are personalized to each user

Similar to our goals with **bandit algorithms**

- **Exploit** by recommending content that we user is likely to enjoy (personalization)
- **Explore** by recommending a variety of content (coverage)
1. Help to **filter** huge amounts of content, so that users see content that is **relevant**
2. Maximize **coverage** so that a variety of different content is recommended.
3. Make recommendations that are personalized to each user.
1. Data and problem setting

- **Data:** Blogs ("the blogosphere")

- **Comparison:** other systems that aggregate blog data
1. Data and problem setting

- **Low-level features:**
  Bags-of-words, noun phrases, named entities

- **High-level features:**
  Low-dimensional document representations, topic models
2. Maximize **coverage**

- **We’d like to choose a (small) set of documents that maximally cover the set of features the user is interested in (later)**

**cover** (\(f\)) = amount by which \(\{\text{posts}, \text{set of features}\}\) covers

\(\text{cover}_{A}(f)\) = amount by which \(\{\text{set of documents}\}\) covers

Set \(A\)

Feature \(f\)
2. Maximize coverage

• Can be done (approximately) by selecting documents greedily (with an approximation ratio of \((1 - 1/e)\)
2. Maximize **coverage**

---

Hamas announces ceasefire after Israel declares truce

What are these? Hamas said today it would cease fire immediately along with other militant groups in the Gaza Strip and give Israel, which already declared a unilateral truce, a week to pull its troops out of the territory. A spokesman for Israeli Prime Minister Ehud Olmert said earlier that if a c...

---

**from SEMISSOURIAN.COM**

Warner leads Cardinals to first Super Bowl appearance

By BARRY WILNER The Associated Press Arizona Cardinals defensive end Calais Campbell celebrates after the NFL NFC championship football game against the Philadelphia Eagles Sunday, Jan. 18, 2009, in Glendale, Ariz. The Cardinals won 32-25...

---

**from NORTHJERSEY.COM**

Stars, throngs shine as D.C. opens Inaugural celebrations

Last updated: Monday January 19, 2009, 8:47 AM A who's who of movie and musical stars joined President-elect Barack Obama on Sunday for an opening celebration of the run-up to Inau...

---

**from CBS5.COM**

President-Elect Barack Obama Honors Martin Luther King Jr. On Obama Visits Troops, Shelter, Honors MLK Jr. Jan 19, 2009 6:03 PM

---

**from CTV**

Plane's recorders capture sudden loss of engine power

---

Works pretty well! (and there are some comparisons to existing blog aggregators in the paper) **But** – no personalization
3. Personalize

$$F(A) = \sum_{f \in U} \pi_{u,f} \cdot w_f \cdot cover_A(f)$$

- Need to learn weights for each user based on their feedback (e.g. click/not-click) on each post
3. Personalize

\[ F(\mathcal{A}) = \sum_{f \in \mathcal{U}} \pi_{u,f} \cdot w_f \cdot \text{cover}_\mathcal{A}(f) \]

- Need to learn weights for each user based on their feedback (e.g. click/not-click) on each post
  - A click (or thumbs-up) on a post increases \( \pi_{u,f} \) for the features \( f \) associated with the post
  - Not clicking (or thumbs-down) decreases \( \pi_{u,f} \) for the features \( f \) associated with the post
3. Personalize

- Feedback on articles suggested
- Weighted interest in topic

Day 1

Day 2

Day 3
Summary

• Want an algorithm that **covers** the set of topics that each user wants to see
• Articles can be chosen **greedily**, while still covering the topics nearly optimally
• The topics to cover can also be **personalized** to each user, by updating their preferences in response to user feedback
• **Evaluated** on real blog data (see paper!)
We’ve looked at three features to handle the properties unique to online advertising

1. We need to handle **budgets** at the level of users and content (Matching problems)
2. We need algorithms that can operate **online** (i.e., as users arrive one-at-a-time) (AdSense)
3. We need to algorithms that exhibit an explore-exploit tradeoff (Bandit algorithms)
Further reading:

• Turning down the noise in the blogosphere (by Khalid El-Arini, Gaurav Veda, Dafna Shahaf, Carlos Guestrin)
Assignment 1
Assignment 1