Announcements

- Assignment 0 is due on wed.
- Read Chapters 1 & 2 of Forsyth & Ponce
- Lecture notes on web page
- Waitlist
- Two slide decks that might be useful:
  - Linear Algebra Review
  - Probability and Random Variables Review
- (Subset of?) Final exam can be use for CSE MS Comprehensive Exam [Pending]

The course

- Part 1: The physics of imaging
- Part 2: Early vision
- Part 3: Reconstruction
- Part 4: Recognition

Image Formation: Outline

- Factors in producing images
- Projection
- Perspective/Orthographic Projection
- Vanishing points
- Projective Geometry
- Rigid Transformation and SO(3)
- Lenses
- Sensors
- Quantization/Resolution
- Illumination
- Reflectance and Radiometry

Earliest Surviving Photograph

- First photograph on record, “la table service” by Nicephore Niepce in 1822.
- Note: First photograph by Niepce was in 1816.

Compare to Paintings

Willem Kalf, Mid 1600’s

Pedro Campos,
How Cameras Produce Images

- Basic process:
  - photons hit a detector
  - the detector becomes charged
  - the charge is read out as brightness

- Sensor types:
  - CCD (charge-coupled device)
    - high sensitivity
    - high power
    - cannot be individually addressed
    - blooming
  - CMOS
    - simple to fabricate (cheap)
    - lower sensitivity, lower power
    - can be individually addressed

Images are two-dimensional patterns of brightness values.

Effect of Lighting: Monet

Change of Viewpoint: Monet

Haystack at Chailly at sunrise (1865)

Pinhole Camera: Perspective projection

- Abstract camera model - box with a small hole in it

Camera Obscura

“When images of illuminated objects ... penetrate through a small hole into a very dark room ... you will see [on the opposite wall] these objects in their proper form and color, reduced in size ... in a reversed position, owing to the intersection of the rays.” --- Leonardo Da Vinci

http://www.acmi.net.au/AIC/CAMERA_OBSCURA.html (Russell Naughton)
Camera Obscura

- Used to observe eclipses (e.g., Bacon, 1214-1294)
- By artists (e.g., Vermeer).

Jetty at Margate England, 1898.

http://brightbytes.com/cosite/collection2.html (Jack and Beverly Wilgus)

Distant objects are smaller

(Forsyth & Ponce)

Purely Geometric View of Perspective

The projection of the point \( P \) on the image plane \( \Pi' \) is given by the point of intersection \( P' \) of the ray defined by \( PO \) with the plane \( \Pi' \).

Equation of Perspective Projection

Cartesian coordinates:
- We have, by similar triangles, that \((x, y, z) \rightarrow (f' x/z, f' y/z, f')\)
- Establishing an image plane coordinate system at \( C' \) aligned with \( i \) and \( j \), we get \((x, y, z) \rightarrow (f' x/z, f' y/z, f')\)

Geometric properties of projection

- 3-D points map to points
- 3-D lines map to lines
- Planes map to whole image or half-plane
- Polygons map to polygons

- Important point to note: Angles & distances not preserved, nor are inequalities of angles & distances.
- Nor are ratios of angles or distances
- Degenerate cases:
  - line through focal point projects to point
  - plane through focal point projects to a line
A Digression

Projective Geometry and Homogenous Coordinates

Do two lines in the plane always intersect at a point?

No, Parallel lines don’t meet at a point.

Can the perspective image of two parallel lines meet at a point?

YES

Projective Geometry

- Axioms of Projective Plane
  1. Every two distinct points define a line
  2. Every two distinct lines define a point (intersect at a point)
  3. There exists three points, A,B,C such that C does not lie on the line defined by A and B.
- Different than Euclidean (affine) geometry
- Projective plane is “bigger” than affine plane – includes “line at infinity”

Projective Plane = Affine Plane + Line at Infinity
Homogenous coordinates
A way to represent points in a projective space

- Use three numbers to represent a point on a projective plane
  Why? The projective plane has to be bigger than the Cartesian plane.

How: Add an extra coordinate
  e.g., \((x,y) \rightarrow (x,y,1)\)

Impose equivalence relation
  \((x,y,z) \approx \lambda (x,y,z)\)
  such that \((\lambda \neq 0)\)
  i.e., \((x,y,1) \approx (\lambda x, \lambda y, \lambda)\)

- Point at infinity – zero for last coordinate
  e.g., \((x,y,0)\)

- Why do this?
  – Possible to represent points “at infinity”
  – Possible to write the action of a perspective camera as a matrix

Conversion
Euclidean -> Homogenous -> Euclidean

In 2-D
- Euclidean -> Homogenous:
  \((x, y) \rightarrow k (x,y,1)\)
- Homogenous -> Euclidean:
  \((x, y, z) \rightarrow (x/z, y/z)\)

In 3-D
- Euclidean -> Homogenous:
  \((x, y, z) \rightarrow k (x,y,z,1)\)
- Homogenous -> Euclidean:
  \((x, y, z, w) \rightarrow (x/w, y/w, z/w)\)

Projective transformation
- 3 x 3 linear transformation of homogenous coordinates
  Points map to points,
  lines map to lines

\[
\begin{bmatrix}
    u_1 \\
u_2 \\
u_3
\end{bmatrix} =
\begin{bmatrix}
    a_{11} & a_{12} & a_{13} \\
    a_{21} & a_{22} & a_{23} \\
    a_{31} & a_{32} & a_{33}
\end{bmatrix} \begin{bmatrix}
    x_1 \\
x_2 \\
x_3
\end{bmatrix}
\]

Points at infinity
Point at infinity – zero for last coordinate \((x,y,0)\)
and equivalence relation
  \((x,y,0) \approx \lambda (x,y,0)\)

No corresponding Euclidean point

The equation of projection
Cartesian coordinates:

\[
\begin{bmatrix}
    x' \\
y' \\
z'
\end{bmatrix} =
\begin{bmatrix}
    x & y & z & 1
\end{bmatrix}
\]

Homogenous Coordinates

and Camera matrix

\[
\begin{bmatrix}
    1 & a_{11} & a_{12} & a_{13} \\
    0 & a_{21} & a_{22} & a_{23} \\
    0 & a_{31} & a_{32} & a_{33}
\end{bmatrix}
\]

\[
\begin{bmatrix}
    X \\
Y \\
Z
\end{bmatrix} =
\begin{bmatrix}
    x & y & z & 1
\end{bmatrix}
\]
In a perspective image, parallel lines meet at a point, called the vanishing point.

Vanishing points

- A scene can have more than one vanishing point.
- Different directions correspond to different vanishing points.

Vanishing Points

- Vanishing point location: Intersection of line through O parallel to given line(s).
- Parallel lines don’t need to be in the same plane.
- A single line can have a vanishing point.
Vanishing Point

- In the **projective plane**, parallel lines meet at a point at infinity.

- The vanishing point is the perspective projection of that point at infinity, resulting from multiplication by the camera matrix.

Simplified Camera Models

- Perspective

  \[
  \begin{bmatrix}
  u \\
  v \\
  w
  \end{bmatrix} = \begin{bmatrix}
  x \\
  y \\
  z
  \end{bmatrix} + \begin{bmatrix}
  \frac{f_x}{z_0} \\
  \frac{f_y}{z_0} \\
  1
  \end{bmatrix} \begin{bmatrix}
  x \\
  y \\
  z
  \end{bmatrix}
  \]

- Perform a Taylor series expansion about \((x_0, y_0, z_0)\)

  \[
  \begin{bmatrix}
  u \\
  v \\
  w
  \end{bmatrix} \approx \begin{bmatrix}
  x \\
  y \\
  z
  \end{bmatrix} + \begin{bmatrix}
  \frac{f_x}{z_0} \\
  \frac{f_y}{z_0} \\
  0
  \end{bmatrix} \begin{bmatrix}
  x - x_0 \\
  y - y_0 \\
  z - z_0
  \end{bmatrix} + \begin{bmatrix}
  \frac{f_x}{z_0} \\
  \frac{f_y}{z_0} \\
  -f_{xx}/z_0^2
  \end{bmatrix} \begin{bmatrix}
  (x - x_0)^2 \\
  (y - y_0)^2 \\
  (z - z_0)^2
  \end{bmatrix} + \cdots
  \]

- Dropping higher order terms and regrouping.

  \[
  \begin{bmatrix}
  u \\
  v \\
  w
  \end{bmatrix} \approx \begin{bmatrix}
  x \\
  y \\
  z
  \end{bmatrix} + \begin{bmatrix}
  \frac{f_x}{z_0} \\
  \frac{f_y}{z_0} \\
  0
  \end{bmatrix} \begin{bmatrix}
  x - x_0 \\
  y - y_0 \\
  z - z_0
  \end{bmatrix} = Ap + b
  \]

Affine Camera Model

- Take perspective projection equation, and perform Taylor series expansion about some point \((x_0, y_0, z_0)\).
- Drop terms that are higher order than linear.
- Resulting expression is affine camera model

Scaled orthographic projection

Starting with Affine Camera Model, take Taylor series about \((x_0, y_0, z_0) = (0, 0, z_0)\) – a point on the optical axis.

Note: \(f_z_0\) is the scale
The projection matrix for scaled orthographic projection

\[
\begin{pmatrix}
U \\
V \\
W
\end{pmatrix} =
\begin{pmatrix}
f/z_0 & 0 & 0 & 0 \\
0 & f/z_0 & 0 & 0 \\
0 & 0 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
X \\
Y \\
Z \\
T
\end{pmatrix}
\]

- Parallel lines project to parallel lines
- Ratios of distances are preserved under orthographic projection

Other camera models

- Generalized camera – maps points lying on rays and maps them to points on the image plane.

Some Alternative “Cameras”