CSE 20 Fall 2017
Practice for Midterm exam

This practice exam may help you study for the midterm on Tuesday, October 31. I recommend that you also review each HW assignment (by looking at the feedback you received as well as solutions posted on Piazza), examples from class and the relevant textbook sections, and the recommended practice problems from the class website.

1. Algorithms and optimization Let's define a new algorithm for making change using specified coins. We say that the algorithm is correct for $c_1, c_2, \ldots, c_r$ where $c_1 > c_2 > \cdots c_r$, if for any input value n the value given by the number of coins of each denomination output by the algorithm add up to the value n **and** the number of coins used is as small as possible. The idea of this **cautious algorithm** is to use the lowest denomination coins first.

```
procedure cautiousChange(n : a positive integer)
  for i := 0 to r − 1
    d_{r−i} := 0
    while n ≥ c_{r−i}
      d_{r−i} := d_{r−i} + 1
      n := n − c_{r−i}
  return (d_1, . . . , d_r)
```

For this algorithm to be correct, it would need to output the correct numbers of coins of each denomination (value) so that the coins add up to the input n, and so that this output uses the fewest coins possible.

(a) **Trace** this algorithm when the set of denominations of coins is $c_1 = 4$ cents , $c_2 = 3$ cents and the input amount to be changed is $n = 13$ cents.

(b) Explain why this algorithm **is not correct** for the set of denominations of coins $c_1 = 4$ cents, $c_2 = 3$ cents, and $c_3 = 1$ cent. What is the smallest value of n that gives a counterexample to correctness of the algorithm?

(c) Is this algorithm correct for **any** set of denominations of coins? Prove your answer is correct.

2. Number systems and base expansion

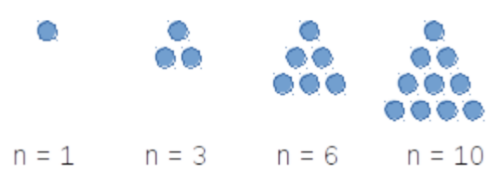
(a) Compute the ternary (base 3) expansion of 28.

(b) Compute the product of $(6A)_{16}$ and $(11)_{16}$, without converting either number to another base.

(c) Confirm your answer for part (b) by converting $(6A)_{16}$ and $(11)_{16}$ to decimal, multiplying them, and converting the product to base 16.

(d) How many bits will there be in the binary (base 2) expansion of 2017? Can you compute this without fully converting 2017 to base 2?

3. Implementing circuits A triangular number (or triangle number) counts the objects that can form an equilateral triangle, as in the diagram below. The n^{th} triangular number is the sum of the first n integers, as shown in the following figure illustrating the first four triangular numbers (what is the fifth one?):



Design a circuit that takes a 4-bit fixed width binary integer $x_3x_2x_1x_0$ as input, and outputs True (T or 1) if this integer is a triangular number, and False (F or 0) otherwise. You may assume that 0 is not a triangular number. (*Credit: UBC Department of Computer Science*)

4. Logical Equivalences

(a) Draw a logic circuit that uses **exactly three** gates and is logically equivalent to

$$q \leftrightarrow (p \wedge r)$$

You may (only) use AND, OR, NOT, and XOR gates.

(b) Write a compound proposition which is logically equivalent to

$$(p \oplus q) \leftrightarrow r$$

You may only use the logical operators negation ($\neg$), conjunction ($\wedge$), and disjunction ($\vee$).

(c) Find a compound proposition that is in DNF (disjunctive normal form) and is logically equivalent to

$$(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee r) \wedge (\neg p \vee q \vee r)$$

5. Logic

p is "The display is 13.3-inch"

q is "The processor is 2.2 GHz"

r is "There is at least 128GB of flash storage"

s is "There is at least 256GB of flash storage"

u is "There is at least 512GB of flash storage"

(a) Are the statements

$$p \rightarrow (r \vee s \vee u) \quad , \quad q \rightarrow (s \vee u) \quad , \quad p \leftrightarrow q \quad , \quad \neg u$$

consistent? If so, translate to English a possible assignment of truth values to the input propositions that makes all four statements true simultaneously.

(b) Consider this statement in English:

It's not the case that both the display is 13.3-inch and the processor is 2.2 GHz.

Determine whether each of the compound propositions below is equivalent to the negation of that statement, and justify your answers using either truth tables or other equivalences.

Possible compound propositions:

(I) $\neg p \vee \neg q$ (II) $\neg(p \rightarrow \neg q)$ (III) $\neg(p \wedge q)$ (IV) $(\neg p \leftrightarrow \neg q) \wedge p$

(c) Consider the compound proposition

$$(p \wedge q) \rightarrow (r \vee s \vee u)$$

Express the **contrapositive** of this conditional as a compound proposition.

Then, give an assignment of truth values to each of the input propositional variables for which the original compound proposition is True but its **converse** is False.

6. Logical equivalence

(a) Over the domain $\{1, 2, 3, 4, 5\}$ give an example of predicates $P(x), Q(x)$ which demonstrate that

$$\forall x P(x) \vee \forall x Q(x) \not\equiv \forall x(\ P(x) \vee Q(x)\ )$$

(b) Over the domain $\mathbb{R}$ give an example of predicates $P(x), Q(x)$ which demonstrate that

$$\exists x P(x) \wedge \exists x Q(x) \not\equiv \exists x(\ P(x) \wedge Q(x)\ )$$

7. Proof strategies For each of the following statements, write its negation in clear and precise English. Then decide whether the original statement or its negation is true, and prove it.

(a) There are four consecutive natural numbers whose sum is not divisible by 4.

(b) Over nonnegative integers, $\exists x \forall y((y > 0) \rightarrow (2x < y))$

(c) If x is irrational, then so is $2x$.

(d) Every perfect square is the sum of two (not necessarily distinct) perfect squares.