HW1 graded – review form?
HW2 released

CSE 20
DISCRETE MATH

Fall 2017

http://cseweb.ucsd.edu/classes/fa17/cse20-ab/

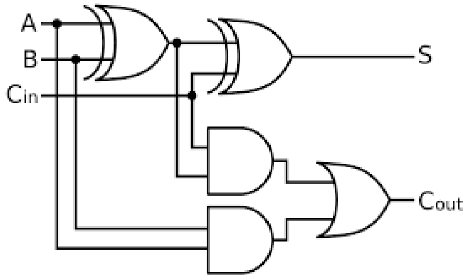
Today's learning goals

- Translate sentences from English to propositional logic using appropriate propositional variables and boolean operators.
- Truth tables: negation, conjunction, disjunction, exclusive or, conditional, biconditional operators.
- Evaluate the truth value of a compound proposition given truth values of its constituent variables.
- Form the converse, contrapositive, and inverse of a given conditional statement.
- Decide and justify whether or not a collection of propositions is consistent.

Logic

Rosen Section 1.1

- Use gates and circuits to express arithmetic.



- Precisely express theorems and invariant statements.
- Make valid arguments to prove theorems.

Circuits → Propositions

- 0 (off) → False
- 1 (on) → True

p	q	p ∨ q	p ∧ q	p ⊕ q
T	T	T	T	F
T	F	T	F	T
F	T	T	F	T
F	F	F	F	F

(Order switched!)

Definitions

Rosen p. 2-4

- **Proposition**: declarative sentence that is T or F (not both)
- **Propositional variable**: variables that represent propositions.
- **Compound proposition**: new propositions formed from existing propositions using logical operators.
- **Truth table**: table with 1 row for each of the possible combinations of truth values of the input and an additional column that shows the truth value of the result of the operation corresponding to a particular row.

Truth table: (p v q) v (p v r)

How many rows are in the truth table for **(p v q) v (p v r)**?

A. 1
B. 2
C. 3
D. 4
E. None of the above

$2^3 = 8$

use brackets!

How do we fill in these rows?

- Inputs
- Output

Precedence order of operations on p. 11

Truth tables

- Can use truth table to compute truth value of compound proposition.
- Also, can specify logical operator by truth table.

Truth tables

Can use truth table to compute value of compound proposition.

p	q	r	(p v q) v (p v r)
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

Compound propositions

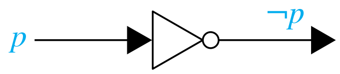
Rosen p. 3-4, 21

p	¬ p
T	F
F	T

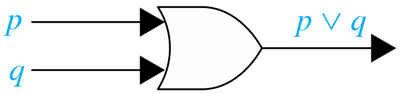
Negation

p	q	p ∨ q p OR q	p ∧ q p AND q	p ⊕ q p XOR q
T	T	T	T	F
T	F	T	F	T
F	T	T	F	T
F	F	F	F	F

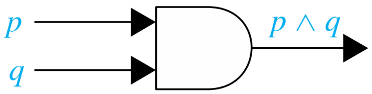
Disjunction Conjunction



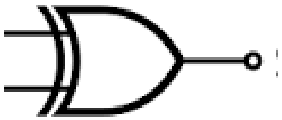
Inverter



OR gate



AND gate



XOR gate

Compound propositions

Rosen p. 10

NOTE $\neg\neg p \equiv p$

Consider the compound proposition

$$\neg(\neg p \vee \neg q)$$

p	q	$\neg(\neg p \vee \neg q)$
T	T	$\neg(\neg T \vee \neg T) =$? $\neg(F \vee F) = \neg F = T$
T	F	$\neg(\neg T \vee \neg F) =$? $\neg(F \vee T) = \neg T = F$
F	T	? F
F	F	? F

To fill in rows

Plug in values one row at a time.

OR

Use intermediate

Compound propositions

Rosen p. 10

Consider the compound proposition

$$\neg(\neg p \lor \neg q)$$

p	q	$\neg p$	$\neg q$	$\neg p \lor \neg q$	$\neg(\neg p \lor \neg q)$
T	T				?
T	F				?
F	T				?
F	F				?

Compound propositions

Rosen p. 10

Consider the compound proposition

$$\neg(\neg p \vee \neg q)$$

p	q	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$\neg(\neg p \vee \neg q)$
T	T	F	F	F	**T**
T	F	F	T	T	**F**
F	T	T	F	T	**F**
F	F	T	T	T	**F**

Does this look familiar?

Logical equivalences

Rosen p. 25

Compound propositions that have the same truth values in all possible cases are **logically equivalent**, denoted $\equiv$.

p	q	$\neg(\neg p \vee \neg q)$
T	T	T
T	F	F
F	T	F
F	F	F

input output

$p \wedge q$: T, F, F, F output

What compound proposition is logically equivalent to $\neg(\neg p \vee \neg q)$?

A. $p \wedge q$
B. $p \vee q$
C. $p \wedge \neg p$
D. $q \vee \neg q$
E. None of the above.

Tautology and contradiction

Rosen p. 25

Tautology: compound proposition that is always T

Contradiction: compound proposition that is always F

p	q	F	T
T	T	F	T
T	F	F	T
F	T	F	T
F	F	F	T

Which of the following is a tautology?

A. p

B. $p \vee p$

C. $p \wedge p$

D. $p \vee \neg p$

E. $p \wedge \neg p$

Tautology and contradiction

Rosen p. 25

Tautology: compound proposition that is always T

Contradiction: compound proposition that is always F

p	q	F	T
T	T	F	T
T	F	F	T
F	T	F	T
F	F	F	T

Are all compound propositions either tautologies or contradictions?

Contingencies

Translation

Rosen p. 22: 1.2#7

Lots of examples here!

Express the sentence "The message was sent from an unknown system but it was not scanned for viruses" using the propositions

p: "The message is scanned for viruses"
q: "The message was sent from an unknown system"

A. $p \wedge q$
B. $p \wedge \neg q$
C. $\neg p \vee q$
D. $p \vee \neg q$
E. None of the above.

F. $\neg p \wedge q$

"implicit and"

q but $\neg p$

i.e. $q \wedge \neg p$.

System specification + consistency

Rosen p. 18

Can such a system be built?

System specifications are **consistent** if they do not contain conflicting requirements

In other words: the specifications are consistent if there is a truth assignment to the input propositional variables that makes each specification true.

System specification + consistency

Rosen p. 18

System specifications are **consistent** if they do not contain conflicting requirements

Practically speaking

Start with system specifications in English

Translate to compound propositions

Fill in truth table with one column for each of the specifications

Look for row in truth table where ***each output column*** evaluates to T

Conditionals

Rosen p. 6-10

Hypothesis

Conclusion

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

$\equiv \neg(p \wedge \neg q)$

Vacuously true

"If p, then q"

p	q	$\neg p \wedge \neg q$
T	T	F
T	F	F
F	T	F
F	F	T

Conditionals

Rosen p. 6-10

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

"If p, then q"

The only way to make a conditional statement false is to …

Conditionals

Rosen p. 6-10

Hypothesis
Antecedent

Conclusion
Consequent

p	q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

"If p, then q"

If HYP, then CONC

Conditionals

Rosen p. 6-10

Flip order

HYP ↓ CONC

p	q	p → q	q → p	H: $\neg q$ → C: $\neg p$	$\neg p \rightarrow \neg q$
T	T	T	T		
T	F	F	T	F	
F	T	T	F		F
F	F	T	T		

Converse of p→q | **Contrapositive of p→ q** | **Inverse of p→q**

To break promise: $\neg q$ T and $\neg p$ F

i.e. q F and p T

Conditionals

Rosen p. 6-10

p	q	p → q	q → p	$\neg q \rightarrow \neg p$	$\neg p \rightarrow \neg q$
T	T	T			
T	F	F		F	
F	T	T	F		F
F	F	T			

Converse of p→q

Contrapositive of p→ q

Inverse of p→q

Conditionals

Rosen p. 6-10

p	q	p → q	q → p	$\neg q \rightarrow \neg p$	$\neg p \rightarrow \neg q$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

Converse of p→q

Contrapositive of p→ q

Inverse of p→q

Biconditionals

Rosen p. 6-10

Which of these compound propositions **is** logically equivalent to $p \leftrightarrow q$?

A. $p \to q$

B. $p \wedge q$

C. $p \vee q$

D. $p \oplus q$

E. None of the above.

"If and only if"
"Necessary and sufficient"

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Biconditionals

Rosen p. 6-10

"If and only if"
"Necessary and sufficient"

Notice:
Compound propositions A and B are logically equivalent
iff
A←→B is a tautology

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Translation

HYP → CONC

Rosen p. 22: 1.2#7

Express the sentence
"The message is scanned for viruses whenever the message was sent from an unknown system" using the propositions

p: "The message is scanned for viruses"
q: "The message was sent from an unknown system"

A. $p \wedge q$
B. $p \vee q$
C. $p \rightarrow q$
D. $p \leftrightarrow q$
E. None of the above.

if
when
p whenever q

$q \rightarrow p$

System specification + consistency

Rosen p. 23 #11

The router can send packets to the edge system only if it supports the new address space.

For the router to support the new address space, it is necessary that the latest software release be installed.

The router can send packets to the edge system if the latest software release is installed.

The router supports the new address space.

p : The router can send packets

System specification + consistency

Rosen p. 23 #11

p only if q.

For q, it is necessary that r.

p if r.

q.

System specification + consistency

Rosen p. 23 #11

p only if q. $\neg q \to \neg p$

For q, it is necessary that r. $\neg r \to \neg q$

p if r. $r \to p$

q. q

System specification + consistency

Rosen p. 18

p only if q. $\neg q \rightarrow \neg p$

For q, it is necessary that r. $\neg r \rightarrow \neg q$

p if r. $r \rightarrow p$

q. q

Can such a system be built?

System specifications are **consistent** if they do not contain conflicting requirements

In other words: the specifications are consistent if there is a truth assignment to the input propositional variables that makes each specification true.

System specification + consistency

Rosen p. 23 #11

p	q	r	$\neg q \rightarrow \neg p$	$\neg r \rightarrow \neg q$	$r \rightarrow p$	q
T	T	T	T	T	T	T
T	T	F	T	F	T	T
T	F	T	F	T	T	F
T	F	F	F	T	T	F
F	T	T	T	T	F	T
F	T	F	T	F	T	T
F	F	T	T	T	F	F
F	F	F	T	T	T	F

System specification + consistency

Rosen p. 18

p	q	r	$\neg q \to \neg p$	$\neg r \to \neg q$	$r \to p$	q
T	T	T	T	T	T	T
T	T	F	T	F	T	T
T	F	T	F	T	T	F
T	F	F	F	T	T	F
F	T	T	T	T	F	T
F	T	F	T	F	T	T
F	F	T	T	T	F	F
F	F	F	T	T	T	F

System specifications are **consistent** if they do not contain conflicting requirements

Reminders

- Discussion section tomorrow
- Review quiz "due" tomorrow
- HW2 due Friday