Thank you to the TAs + tutors for a speedy grading marathon!

CSE 20
DISCRETE MATH

Fall 2017

http://cseweb.ucsd.edu/classes/fa17/cse20-ab/

Today's learning goals

- Correctly prove statements using appropriate style conventions, guiding text, notation, and terminology
- Define and differentiate between important sets
- Use correct notation when describing sets: {...}, intervals
- Define and prove properties of: subset relation, power set, Cartesian products of sets, union of sets, intersection of sets, disjoint sets, set differences, complement of a set

Overall strategy

- Do you believe the statement? its negation?
  - Try some small examples.
- Determine logical structure + main connective.
- Determine relevant definitions.
- Map out possible proof strategies.
  - For each strategy: what can we **assume**, what is the **goal**?
  - Start with simplest, move to more complicated if/when get stuck.

Direct proof, construction, exhaustive, etc.

Contradiction, hidden cases

Some definitions

Rosen Sections 2.1, 2.2

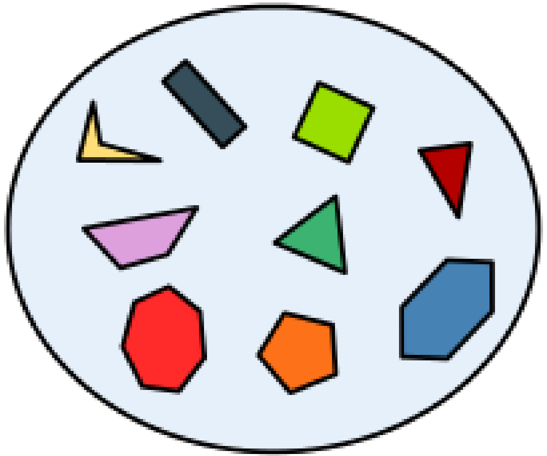
Set: unordered collection of **elements**

is an element of

A = B iff $\forall x (x \in A \leftrightarrow x \in B)$

How to specify these elements?

- Roster $\{ \ldots \}$
- Set builder $\{x \in U \mid P(x)\}$



$\boxed{\text{Proper}}$ subset

$A \subsetneq B$ means $A \subseteq B \wedge A \neq B$

Some definitions

Rosen Sections 2.1, 2.2

"A is a subset B"

Subset: $A \subseteq B$ means $\forall x (x \in A \rightarrow x \in B)$

For sets A and B, A = B if and only if both

$A \subseteq B$ and $B \subseteq A$

Why a definition for equality?

$$\left\{ \frac{p}{q} \mid p, q \in \mathbb{Z}, q \neq 0 \right\} \quad , \quad \left\{ \frac{p}{q} \mid p, q \in \mathbb{Z}, q > 0, \gcd(p, q) = 1 \right\}$$

Some definitions

Rosen Sections 2.1, 2.2

Subset: $A \subseteq B$ means $\forall x (x \in A \rightarrow x \in B)$

$\forall A \, \forall B \left[(A = B) \longleftrightarrow (A \subseteq B \land B \subseteq A) \right]$

all

Theorem: For sets A and B, A = B if and only if both

$A \subseteq B$ and $B \subseteq A$

Proof: Let A, B be arbitrary sets. WTS

(1) If $A = B$ then $A \subseteq B$ and $B \subseteq A$, (2) if $A \subseteq B$ and $B \subseteq A$, then $A = B$.

What's the logical structure of this statement?

A. Universal conditional.

B. Biconditional. universal

C. Conjunction (and)

D. None of the above.

Some definitions

Rosen pp. 119-120 Sections 2.1

$A \not\subseteq B$ means $\neg\forall x(x \in A \to x \in B) \equiv \exists x(x \in A \land x \notin B)$

Subset: $A \subseteq B$ means $\forall x(x \in A \to x \in B)$

Proper subset means $A \subseteq B$ and $A \neq B$ denoted $A \subsetneq B$

When A is a subset of B, we say B is a superset of A. denoted $B \supseteq A$

Which is **not** true?

~~A.~~ $\boxed{\textbf{Z}^{+} \text{ is a subset of } \textbf{Z}.}$

~~B.~~ **Z** is a subset of **Q**.

~~C.~~ **Q** is a subset of **R**.

~~D.~~ **R** is not a subset of **Q**.

For any arbitrary element, if it's in $\mathbb{Z}^+$ then it's a pos int so it's an int, so it's in $\mathbb{Z}$.

Pf There is an example, $\sqrt{2}$, of an element of $\mathbb{R}$ that is not in $\mathbb{Q}$

Bonus Question: Is $\mathbb{Z} = \{\mathbb{Z}\}$?

Some definitions

Rosen Sections 2.1, 2.2

Empty set: $\emptyset = \{\} = \{x \mid x \neq x\}$

Which of the following is **not** true?

A. $\emptyset \subseteq \emptyset$ is true

B. $\emptyset \in \emptyset$ not true b/c $\emptyset$ has no elements.

C. For any set A, $\emptyset \subseteq A$ is true

D. For some set B, $\emptyset \in B$ construction: $B = \{\emptyset, \mathbb{Z}, 1, \pi\}$

E. More than one of the above.

For all sets B, $\emptyset \in B$ F by counterexample $\emptyset$ or $\{1\}$

Operations on sets

"the set of"

Rosen Sections 2.1, 2.2

Power set: For a set S, its power set is the set of all subsets of S. $\mathcal{P}(S) = \{A \mid A \subseteq S\}$

"such that"

$\mathcal{P}(\{2\})$

$\{2\} \subseteq \{2\}$

$\emptyset = \{\ \} \subseteq \{2\}$

$\mathcal{P}(\{2\}) = \{ \emptyset, \{2\} \}$

Operations on sets

Rosen Sections 2.1, 2.2

Power set: For a set S, its power set is the set of all subsets of S. $\mathcal{P}(S) = \{A \mid A \subseteq S\}$

Which of the following is **not** necessarily (always) true?

A. $\boxed{S \in \mathcal{P}(S)}$ WTS $S \subseteq S$ i.e. WTS $\forall x \, (x \in S \to x \in S)$ T

B. $\emptyset \in \mathcal{P}(S)$ WTS $\emptyset \subseteq S$ T

C. $\emptyset \subseteq \mathcal{P}(S)$

D. $\emptyset \in S$ ex. $S = \{1\}$ $\emptyset \notin S$ ex. $\mathbb{Z}$ $\emptyset \in \mathbb{Z}$.

E. None of the above.

Operations on sets

Rosen Sections 2.1, 2.2

Given two sets A, B we can define

$A \cap B$ **Intersection of A and B**

$A \cup B$ **Union of A and B**

$A - B$ **Difference of A and B**

$A \times B$ **Cartesian product of A and B**

E even integers   O odd integers

Operations on sets

Rosen Sections 2.1, 2.2

Given two sets A, B we can define

$$A \cap B = \{x \mid x \in A \wedge x \in B\}$$

$$A \cup B = \{x \mid x \in A \vee x \in B\}$$

$$A - B = \{x \mid x \in A \wedge x \notin B\}$$

$$A \times B = \{(x, y) \mid x \in A \wedge y \in B\}$$

Examples

Operations on sets

Rosen Sections 2.1, 2.2

Given two sets A, B we can define

$$A \cap B = \{x \mid x \in A \wedge x \in B\}$$

$$A \cup B = \{x \mid x \in A \vee x \in B\}$$

$$A - B = \{x \mid x \in A \wedge x \notin B\}$$

$$A \times B = \{(x, y) \mid x \in A \wedge y \in B\}$$

Which of these **is** true?

A. $A \cap B = B \cap A$

B. $A \cup B = B \cup A$

C. $A - B = B - A$

D. $A \times B = B \times A$

E. None of the above.

Operations on sets

Rosen Sections 2.1, 2.2

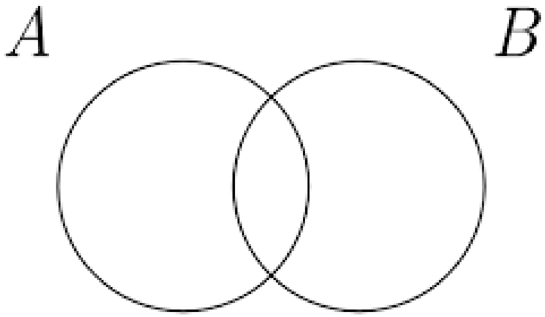
Given two sets A, B we can define

$$A \cap B = \{x \mid x \in A \wedge x \in B\}$$

$$A \cup B = \{x \mid x \in A \vee x \in B\}$$

$$A - B = \{x \mid x \in A \wedge x \notin B\}$$

$$A \times B = \{(x, y) \mid x \in A \wedge y \in B\}$$



Which of the following **can't** be labelled in this Venn diagram?

A. $A \cap B$

B. $A \cup B$

C. $A - B$

D. $A \times B$

E. None of the above.

Operations on sets

Rosen Sections 2.1, 2.2

Given two sets A, B we can define

$$A \cap B = \{x \mid x \in A \wedge x \in B\}$$

$$A \cup B = \{x \mid x \in A \vee x \in B\}$$

$$A - B = \{x \mid x \in A \wedge x \notin B\}$$

When can you conclude that A is a **subset** of B?

A. When $A \cup B = A$
B. When $A - B = A$
C. When $A - B = B - A$
D. When $A \cap B = A$
E. None of the above.

Some definitions

Rosen p. 128 2.2

Disjoint sets: two sets are disjoint if their intersection is the empty set.

Examples?

$$\{1, 2, 3\} \cap \{4, 5, 6\}$$

$$= \emptyset$$

if $A \cap B = \emptyset$ then $A - B = A$ and $B - A = B$

Some definitions

Rosen p. 128 2.2

Disjoint sets: two sets are disjoint if their intersection is the empty set

Which of the following is **not** true?

A. $\emptyset \subseteq \emptyset$

B. $\emptyset \in \emptyset$

C. For any set A, $\emptyset \subseteq A$

D. For some set B, $\emptyset \in B$

E. More than one of the above.

Generalized union / intersection *Rosen p. 132*

Union and intersection are associative

the union of sets A_i as i ranges from 1 to n

$$A_1 \cup A_2 \cup \cdots \cup A_n = \bigcup_{i=1}^{n} A_i$$

$\{1\} \cup \{1,2\} \cup \{1,2,3\} \cup \cdots \cup \{1,2,\ldots,n\} = \{1,2,\ldots,n\}$

$$A_1 \cap A_2 \cap \cdots \cap A_n = \bigcap_{i=1}^{n} A_i$$

$\{1\}$

Analogue: Summation notation

$$a_1 + a_2 + \cdots + a_n = \sum_{i=1}^{n} a_i = \sum_{j=1}^{n} a_j$$

Index of summation

Lower limit

Upper limit

Ex (1): Write an expression for the sum of the first n positive integers.

Ex (2): Write an expression for the sum of the first n positive even integers.

Ex (3): Write an expression for the sum of the first n positive odd integers.