CSE 20 Homework 5
Due: Saturday November 11, 2017 at 11pm

TOPICS Sets and Induction

READING Rosen Sections 1.8, 2.1, 2.2, 2.4, 5.1, 5.2.

KEY CONCEPTS Proof strategies, sets, member / element, subset, empty set, power set, Cartesian product, finite set, union, intersection, set difference, cardinality, mathematical induction.

---

1. (**12 points**) For each pair of sets, determine whether they are **disjoint**, **equal**, **proper subset / superset**, or **none of the above**. Prove your answers.

(a) $\{x \in \mathbb{Z} \mid x^2 = x\}$ and $\{x \in \mathbb{R} \mid x^2 = x\}$.

(b) $\{x \in \mathbb{R} \mid \lfloor x \rfloor = x\}$ and $\mathbb{Q}$.

(c) $\{x \in \mathbb{R} \mid x^2 < x\}$ and $\{x \in \mathbb{R} \mid x \notin \mathbb{Q}\}$.

(d) $\{x \in \mathbb{Z}^+ \mid x \text{ is prime}\}$ and $\{-1, 0, 1\}$. *Note: use the definition of prime as giving in Definition 1 of Section 4.3 "An integer p greater than 1 is prime iff the only positive factors of p are 1 and p."*

2. (**12 points**) For each of the following, determine whether it is true or false and then prove it (if it true) or disprove it (if it is false). In grading this question, we will check whether you correctly determined the truth value for **each** statement, but we may not grade all four proofs.

(a) For sets A, B, if $A \subseteq \mathbb{R}$ and $B \subseteq \mathbb{R}$ and both are closed under addition then $A \cap B$ is also closed under addition.

(b) For sets A, B, if $A \subseteq \mathbb{R}$ and $B \subseteq \mathbb{R}$ and both are closed under addition then $A \cup B$ is also closed under addition.

(c) For A a set,

$$\forall x \left((\{x\} \in \mathcal{P}(A)) \leftrightarrow (x \in A) \right)$$

(d) For sets A, B, C if $A \cap C = B \cap C$ then $A = B$.

3. (**10 points**)

(a) What's wrong with the following reasoning? Clearly articulate and explain any and all errors you find.

$$\left(\sum_{i=1}^{n} a_i\right)\left(\sum_{j=1}^{n} \frac{1}{a_j}\right) = \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{a_i}{a_j} = \sum_{k=1}^{n}\sum_{k=1}^{n} \frac{a_k}{a_k} = \sum_{k=1}^{n} n = n^2.$$

(b) Let b be an arbitrary positive integer greater than 1. Write down a closed-form formula for

$$\sum_{i=0}^{k-1} (b-1)b^i$$

and prove **by induction** that it works for all integers $k \geq 1$. *Hint: you can use properties of number representations to come up with the formula but the proof must still be by induction.*

4. (**6 points**) For $n \geq 2$, a **complete graph** on n vertices (nodes) is a simple graph that contains exactly one edge between each pair of distinct vertices. Prove by **induction** that the number of edges in a complete graph on n vertices is given by the formula

$$\frac{n(n-1)}{2}.$$